Introduction to Neural Networks and Deep Learning

Deep Forward Neural Networks

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August 22, 2020

Outline

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For this initial analysis

We will look at the paper by Bengio

- "Learning deep architectures for AI", Foundations and trends in Machine Learning 2, 1 (2009), pp. 1--127.

And for this, we will look at Boolean functions

- After Shanon pointed out the fact they are useful to represent complex problems [1].

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Architecture

A two-layer circuit of logic gates can represent any boolean function [2]

- Any boolean function can be written as a sum of products, disjunctive normal form:
  - AND gates on the first layer with optional negation of inputs,
  - And OR gate on the second layer

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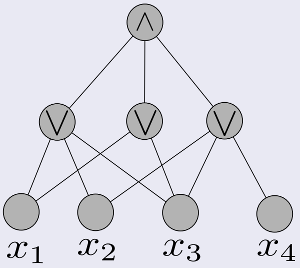

The Exponential Width

Here, we have a small problem

- There are functions computable with a polynomial-size logic gates circuit of depth k that require **exponential size** when restricted to depth $k-1$[3]
  - For Example

$$parity : (b_1, ..., b_d) \in \{0,1\}^d \mapsto \begin{cases} 1 & \text{if } \sum_{i=1}^{d} b_i \text{ is even} \\ -1 & \text{otherwise} \end{cases}$$

How this impact shallow learning in Machine Learning?

- Many of the results for boolean circuits can be generalized to architectures whose computational elements are linear threshold units

$$f(x) = 1_{wx+b>0}$$

  - The fan-in of a circuit is the maximum number of inputs of a particular element.

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How this impact shallow learning in Machine Learning?

- First, we define the concept of f_k function

Definition

- The function f_k is a function of N^{2k-2} variables. It is defined by a depth k circuit that is a tree. At the leaves of the tree there are unnegated variable. The i^{th} level from the bottom consists of $\wedge$-gates if i is even and otherwise it consists of $\vee$-gates.

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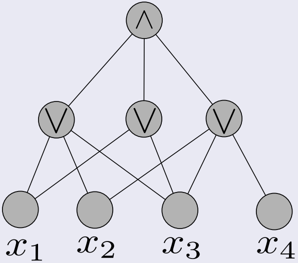

An Important Theorem

Of particular interest is the following theorem

- Monotone weighted threshold circuits (i.e. multi-layer neural networks with linear threshold units and positive weights)

Theorem [4]

- A monotone weighted threshold circuit of depth $k - 1$ computing a function f_k has size at least 2^{cN} for some constant $c > 0$ and $N > N_0$.

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This theorem does not fail any type of architecture

- But the question arises, Are the depth 1, 2 and 3 architectures (many Machine Learning algorithms) too shallow to represent efficiently more complicated functions?

What happens in Deep Architectures

- Bengio et al. argues that they can represent highly-varying functions [5]

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Highly-varying functions

Meaning

- We say that a function is highly-varying when a piecewise approximation of that function would require a large number of pieces.

Clearly

- Deeper Architectures can handle such functions in a easier way than shallow ones.

For Example

- The polynomial $\prod_{i=1}^{n} \sum_{j=1}^{m} a_{ij} x_j$ can be represented as a product of sums with only $O(nm)$ elements

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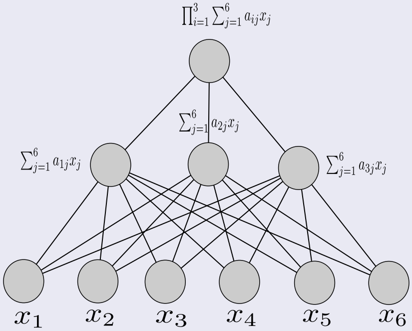
Basically

We have a Perceptron Layer and a Product Second Layer



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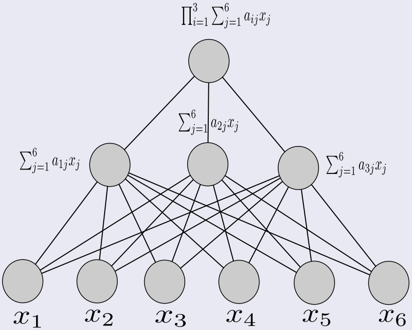

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Because for our case

$$\prod_{i=1}^{3}\sum_{j=1}^{6}a_{ij}x_j=\sum_{j=1}^{6}\prod_{i=1}^{3}a_{ij}x_j$$

We have the following problem $O\left(n^m\right)$

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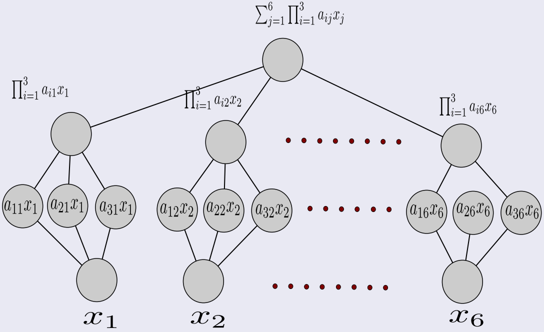

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You could claim

- Machine Learning shallow learning depends on complex computational units to handle complex functions

Deep Learning

- Proposes simpler units but deeper structures to handle complex functions

What about both ideas together

- Complex adaptive units
- Deeper architectures to helps such units
  - It seems to be the case of the human brain...!!!

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Local vs Non-Local Generalization

Something Notable

- A local estimator partitions the input space in regions

Thus, local estimators are based on matching local templates

- It can be thought of as having two levels...

The first level

- It is made of a set of templates which can be matched to the input.

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A template unit will output a value that indicates the degree of matching

$$K\left(x|\Theta\right)$$

The second level combines these values

- Typically a simple linear combination or product combination

$$L\left(x\right)=\sum_{i=1}^{k}K\left(x|\Theta_{i}\right)$$

Classic Example, the kernel machine

$$f\left(x\right)=b+\sum_{i=1}^{k}\alpha_{i}K\left(x,x_{i}\right)$$

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The Kernel has a local influence based on the support vectors

- For example the Gaussian Kernel

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The Problem of Kernel

- The assumption that the target function is smooth or can be well approximated with a smooth function.

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If we have the training example (x_i, y_i)

- We want to build predictor that output something near y_i when any other sample is near x_i

Basically the situation when regularizing

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Although, It is possible to argue

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$$parity : (b_1, ..., b_d) \in \{0,1\}^d \mapsto \begin{cases} 1 & \text{if } \sum_{i=1}^{d} b_i \text{ is even} \\ -1 & \text{otherwise} \end{cases}$$

Theorem

- Let $f(x) = b + \sum_{i=1}^{2^d} \alpha_i K(x_i, x)$ be an affine combination of Gaussian with the same width σ centered on points $x_i \in \{-1,1\}^d$. If f solve the parity problem, then there are at least 2^{d-1} non-zero support vectors.

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Although, this function is not a representative

- The kind of functions we are more interested in AI.

It suggest that local based estimators

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For example

Tensors have been used to add memory to SVM

$$\min_{\boldsymbol{U}_i^{(m)}, \boldsymbol{K}^{(m)}, \boldsymbol{\beta}, b} \gamma \sum_{i=1}^{N} \left\| \mathcal{X}_i - \left[\!\left[\boldsymbol{K}^{(1)} \boldsymbol{U}_i^{(1)}, \cdots, \boldsymbol{K}^{(M)} \boldsymbol{U}_i^{(M)} \right]\!\right] \right\|_F^2 + \cdots$$

$$\lambda \boldsymbol{\beta}^T \widehat{\boldsymbol{K}} \boldsymbol{\beta} + \sum_{i=1}^{N} \left[1 - y_i \left(\widehat{\boldsymbol{k}}_i^T \boldsymbol{\beta} + b \right) \right]_+$$

- $\boldsymbol{K}^{(m)}$ are kernel matrices defined on each mode to capture the nonlinear part.
- $\boldsymbol{U}^{(m)} = \left[\boldsymbol{u}_1^{(m)}, \ldots, \boldsymbol{u}_R^{(m)} \right]$ are factor matrices of size $I_m \times R_m$

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A Problem

- You are limiting the Machine Learning operations to matrix additions and products and non-linear operations.
  - In a shallow way...

We need to add more complex functions

- After all deeper architectures construct complex functions layer by layer

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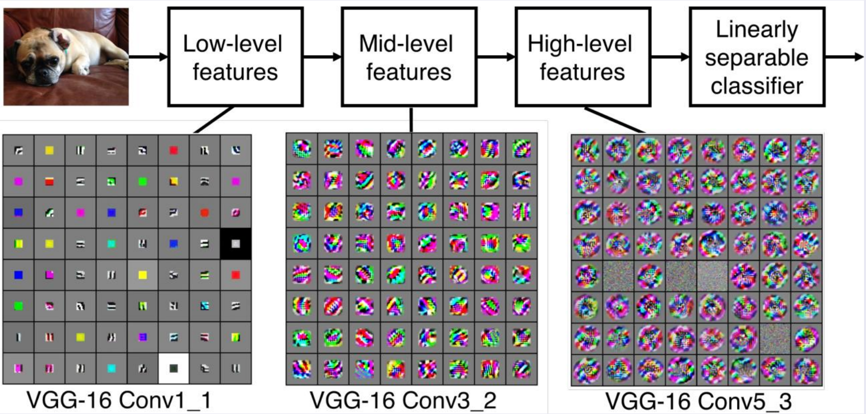
By Using Weights in Certain Deep Learners

The Application of each Layer increase the complexity of the features



Outline

Some of the Models to be Reviewed of Models

Convolutional Neural Networks

- The classic model that started the phenomena of Neural Networks.

Auto Encoder

- How to generate novel features by funneling.

Boltzmann Machine

- Energy Based Models.

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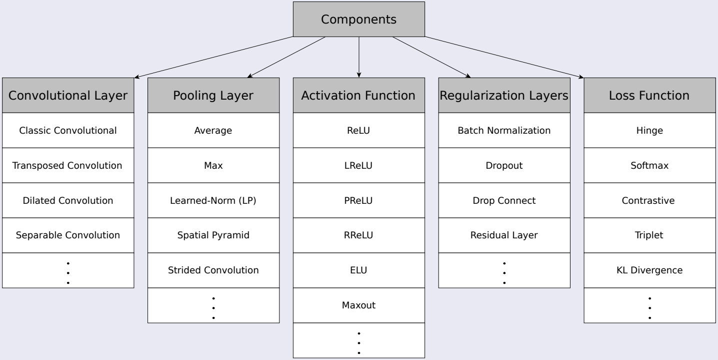
We will see that there are many possible architectures

- And more with the different layers [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] :



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Digital Images as pixels in a digitized matrix

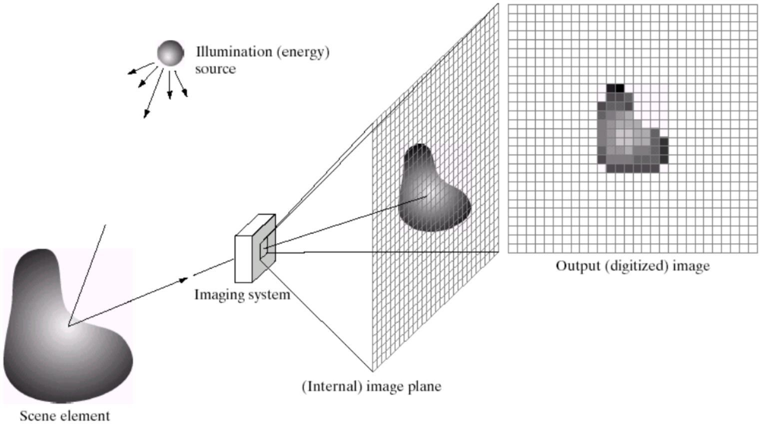

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Image	Noise Removal	Improved Image
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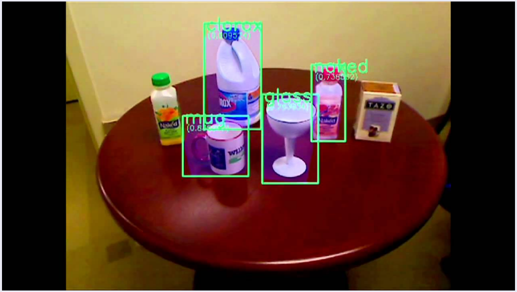

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- We would solve a lot of headaches when setting up such process

Why not to use the data sets

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Work by Hubel and Wiesel in the 1950s and 1960s

- They showed that cat and monkey visual cortexes contain neurons that individually respond to small regions of the visual field.

After all more studies about the visual cortex happened

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Neurocognitron (Circa 1980)

Kunihiko Fukushima [21]

- Proposed a Hierarchical Network for image recognition with a convolution!!!

But it used a function φ

$$\varphi\left(\frac{1+\sum_{k_{l-1}=1}^{K_{l-1}}\sum_{v\in S_l} a_l\left(k_{l-1},v,k_l\right)u_{cl-1}\left(k_{l=1},n+v\right)}{1+\frac{2r_l}{1+r_l}b_l\left(k_l\right)v_{Cl-1}\left(n\right)}-1\right)$$

With a Relu function

$$\varphi\left(x\right)=\begin{cases} x & x\geq 0\\ 0 & x<0\end{cases}$$

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Basically they are deep learners based in convolutions or its variants

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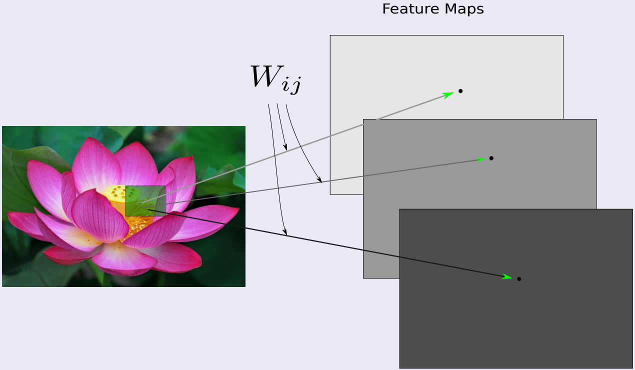
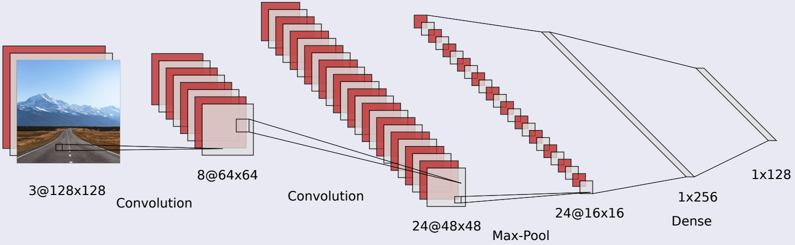


Example of CNN

A Basic Convolutional Network



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We have several techniques for that

Principal Component Analysis

$$L\left(\boldsymbol{u}_1\right) = \boldsymbol{u}_1^T S \boldsymbol{u}_1 + \lambda_1 \left(1 - \boldsymbol{u}_1^T \boldsymbol{u}_1\right)$$

Linear Locally Embeddings

$$\Phi\left(Y\right) = \sum_i \left| Y_i - \sum_j W_{ij} Y_j \right|^2$$

And recently

- Uniform Manifold Approximation and Projection for Dimension Reduction [25]

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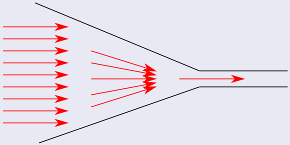
- This can be done by a series of mappings that act as funnels, How?



Basically, we have a series of mappings

$$x \in \mathbb{R}^{n_1} \to f_1(x) \in \mathbb{R}^{n_2} \to f_2(x_1) \in \mathbb{R}^{n_3} \cdots \longrightarrow f_m(x_{m-1}) \in \mathbb{R}^{n_{m+1}}$$

Where

$$n_1 < n_2 < \cdots < n_m < n_{m+1}$$

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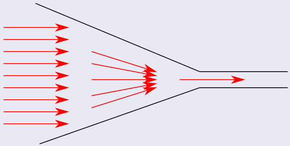

Basically, we have a series of mappings

$$x \in \mathbb{R}^{n_1} \to f_1(x) \in \mathbb{R}^{n_2} \to f_2(x_1) \in \mathbb{R}^{n_3} \cdots \longrightarrow f_m(x_{m-1}) \in \mathbb{R}^{n_{m+1}}$$

Where

$$n_1 < n_2 < \cdots < n_m < n_{m+1}$$

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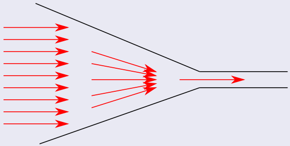

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Then, we can use linear mappings for this

With the following matrix functions

$$\sigma\left[f_{A_{i+1}}\left(x_i\right)\right] = \sigma\left(A_{i+1}x\right)$$

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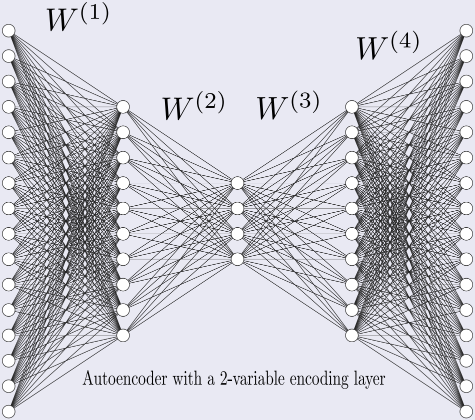
The Basic Auto Encoder Architecture

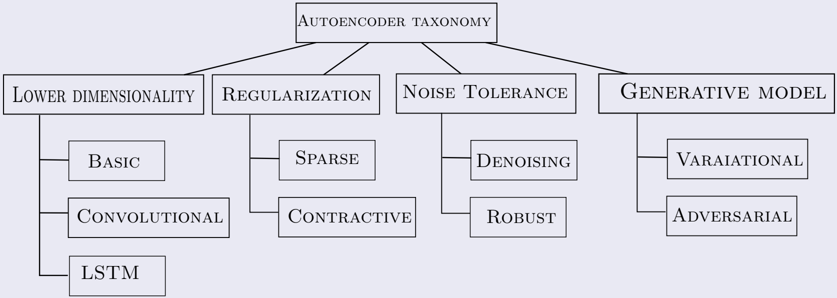
We have



Autoencoder with a 2-variable encoding layer

Taxonomy

Most popular Auto Encoders



Outline

The Basic Energy Models

We have that the Boltzmann Machines

- A Boltzmann machine is a network of units that are connected to each other

Here, we have N be the number of units

- Each unit takes a binary value in $\{0, 1\}$
  - Represented by a random variable X_i, $i = 1, ..., N$.

Additionally, it has parameters

- Bias b_i
- Weight w_{ij} between unit i and unit j, $(i, j) \in [1, N-1] \times [i+1, N]$

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The Energy Based Structure

The energy of the Boltzmann machine is defined by

$$E_{W,\boldsymbol{b}}\left[\boldsymbol{x}\right] = -\sum_{i=1}^{N} b_i x_i - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{ij} x_i x_j = -\boldsymbol{b}^T\boldsymbol{x} - \boldsymbol{x}^T W \boldsymbol{x}$$

This allows to define a probability distribution

$$\mathbb{P}_{W,b}\left(\boldsymbol{x}\right) = \frac{\exp\left(-E_{W,b}\left[\boldsymbol{x}\right]\right)}{\sum_{\widetilde{\boldsymbol{x}}} \exp\left(-E_{W,b}\left[\widetilde{\boldsymbol{x}}\right]\right)}$$

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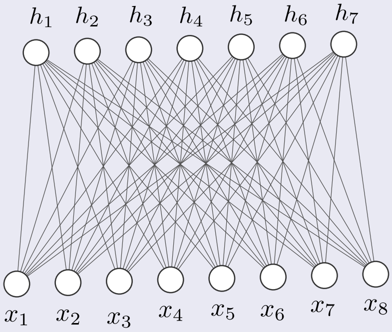
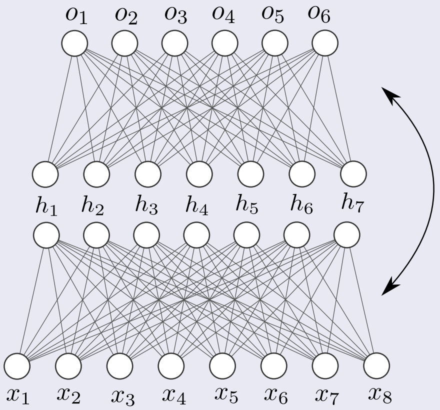
Example

Restricted Boltzmann Machines where the conectivity is layer by layer



Thus, using it as a basic model

We can stack them into a multiple layer model



Outline

Generative Adversarial Networks

They can be seen as an Accept-Reject MCMC Model

- However, they do not require Markov Chains with the classic problem:
  - The independence between the samples to generate ergodic probabilities (The real one)

As in the Accept-Reject

- The **generator network** tries to produce realistic-looking samples
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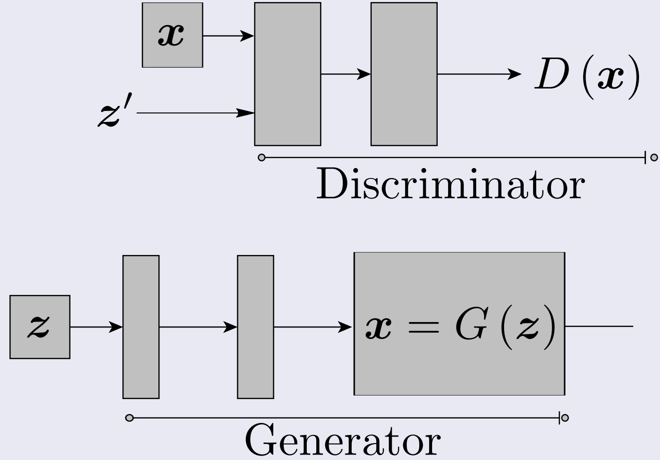
Graphically

We have the following Basic Model



Here

There is a need to join both functions

- So, we can use the idea of Backpropagation to obtain the desired minimization.

How can we do this?

- We can define a sensible learning criterion when the dataset is not linearly separable

For this, we can use the **logistic cross-entropy loss** (We will explain more about this later)

$$\mathcal{L}_{LCE}(z,t) = L_{CE}(\sigma(z), t) = t\log\left(1+e^{-z}\right) + (1-t)\log\left(1+e^{z}\right)$$

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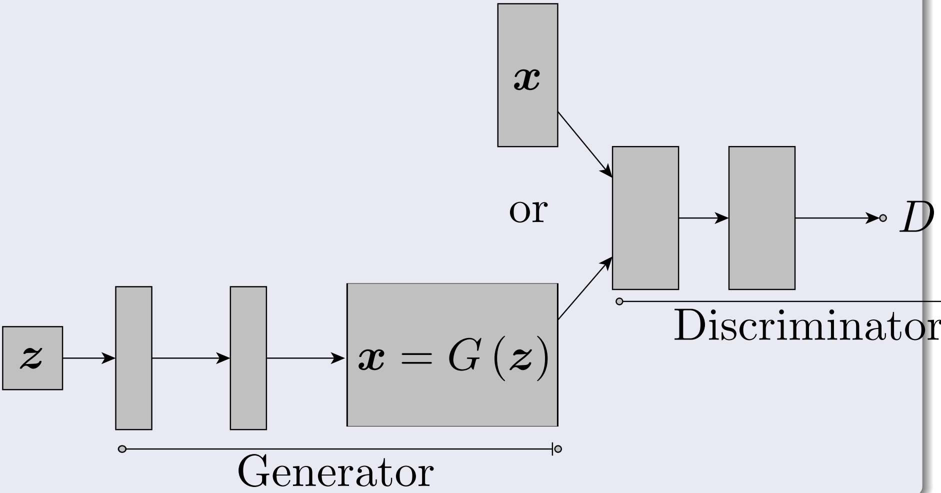
The following architecture use this idea



In this basic Generator

D denote the discriminator's predicted probability of being data

$$\mathcal{J}_D = E_{\boldsymbol{x}\sim\mathcal{D}}\left[-\log D\left(\boldsymbol{x}\right)\right] + E_{\boldsymbol{z}}\left[-\log\left(1 - D\left(G\left(\boldsymbol{z}\right)\right)\right)\right]$$

One possible cost function for the generator

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Then using both functions

The minimax formulation

- Since the generator and discriminator are playing a zero-sum game against each other.

Basically

$$\max_{G} \min_{D} \mathcal{J}_D$$

There are other examples using the LSE [26]

$$\mathcal{J}_G = \frac{1}{N} \sum_{i=1}^{N} \left[G\left(\boldsymbol{z} \right) - \boldsymbol{x} \right]^2$$

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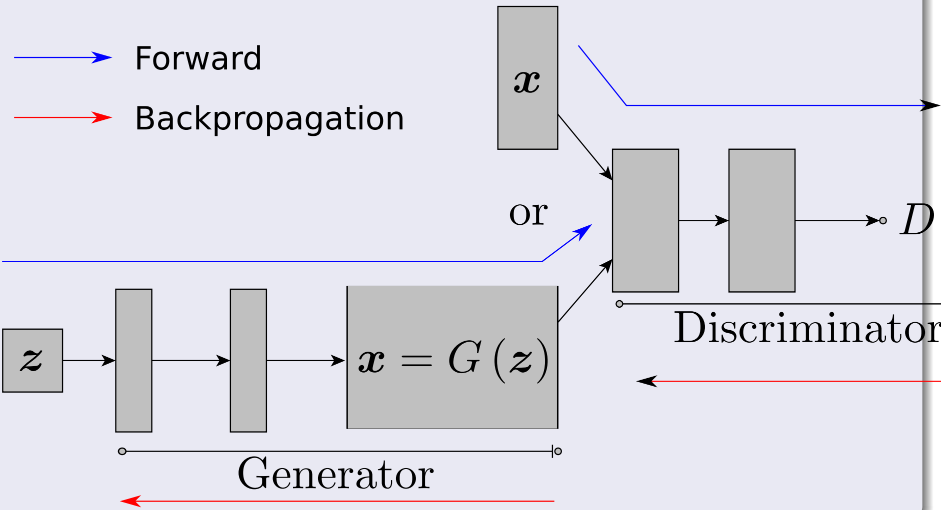
Therefore, we have two updates

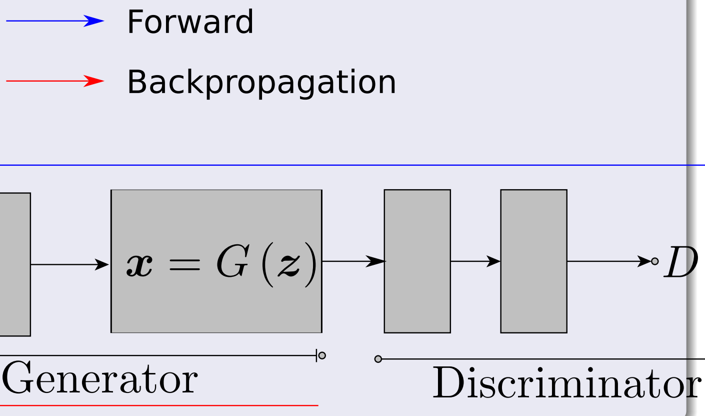
First update the Discriminator



Now

Update the Generator

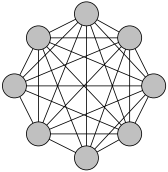
Backprop Derivatives Through the Discriminator, but do not change variables on it... only in the generator



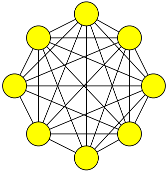
Outline

There Are Many More!!! Here a few more...

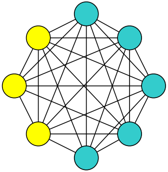
Markov Chain



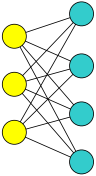
Hopfield Network



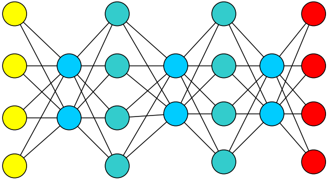
Boltzmann Machine



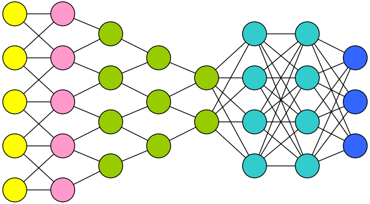
Restricted BM



Deep Belief Network

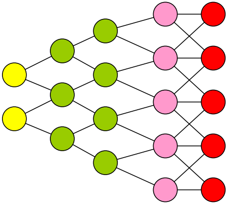


Convolutional Network

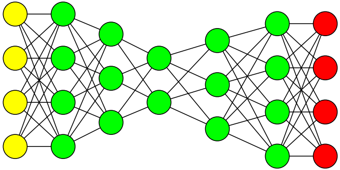


Furthermore

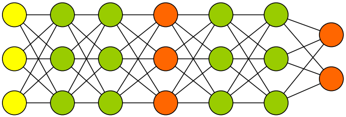
Deconvolutional Network



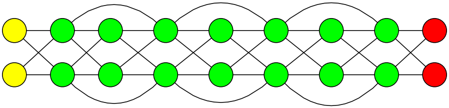
Autoencoder



Generative Adversarial Network



Deep Residual Network



Outline

As We know

In Recurrent Neural Networks, we have the problem

- Vanishing and Exploding Gradients

In the Deeper Architectures as encoder-decoder we have such phenomena

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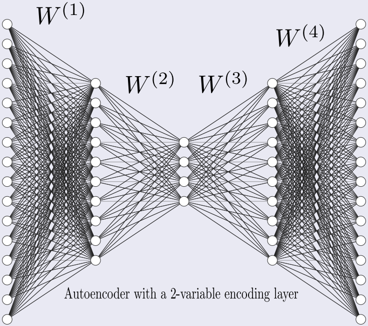
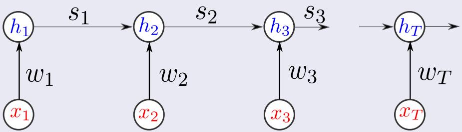


Autoencoder with a 2-variable encoding layer

Consider a simple encoder encoder network

We have this simplified version



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$$h_t = w_t x_t + z_{t-1}$$

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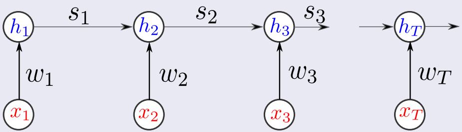

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Backpropagation Rules

Then, we get the following backpropagation rules

$$\frac{\partial h_t}{\partial w_i} = \frac{\partial h_t}{\partial h_{t-1}} \times \frac{\partial h_{t-1}}{\partial h_{t-2}} \times ... \times \frac{\partial h_i}{\partial w_i}$$
$$\frac{\partial h_t}{\partial s_i} = \frac{\partial h_t}{\partial h_{t-1}} \times \frac{\partial h_{t-1}}{\partial h_{t-2}} \times ... \times \frac{\partial h_{i+1}}{\partial s_i}$$

Then, we have

By Using Our simplifying assumption that

$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial \left(w_t x_t + s_{t-1} h_{t-1}\right)}{\partial h_{t-1}} = s_{t-1}$$

And for $\frac{\partial h_i}{\partial w_i}$

$$\frac{\partial h_i}{\partial w_i} = x_t$$

Finally, we have that

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It is clear that

Unless the s_k's are near to 1

- You have the vanishing gradient if $s_k \in [0, 1)$ for all k.
- You have the exploding gradient if $s_k \in (1, +\infty]$ for all k.

Even with activation functions

- These terms tend to appear in the Deep Learners when Backpropagation is done

In the case of Forward

- We have many activation function that squash the signal...

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In the first composition, we get

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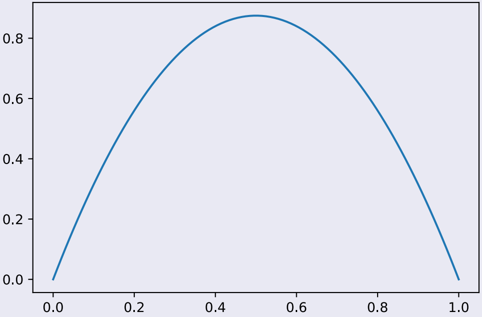
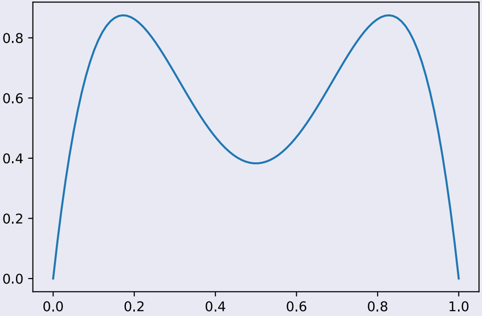


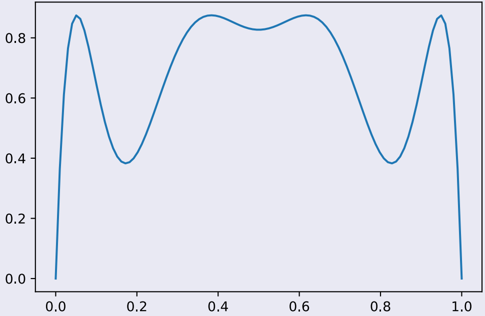
Now, as we compound the function

Second one, $y = f \circ f(x)$



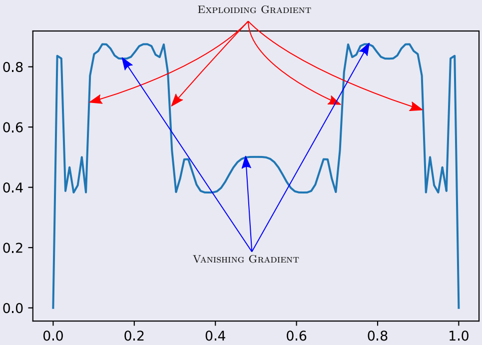
Now, as we increment iterations

Third one, $y = f \circ f \circ f\,(x)$



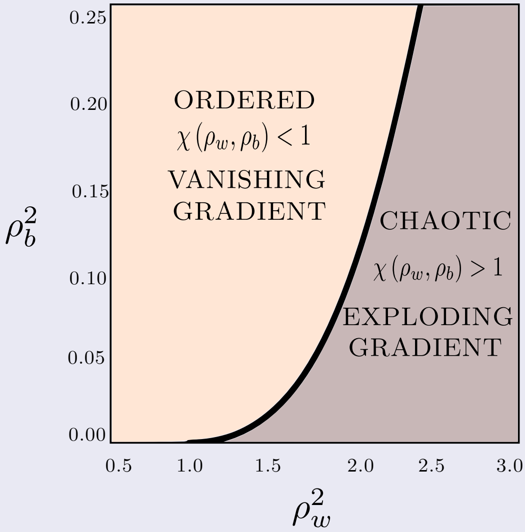
Finally

We see the increment in the gradient part negative or positive



Actually, we have

A Frontier defining the Vanishing and Exploding Gradient [27]



Outline

Actually

Eventually, the iterates go to infinity or zero OR

- They wind up at a fixed point...

A Fixed Point?

$$x = f(x)$$

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A Fixed Point?

$$x = f(x)$$

Basically

The fixed points can be thought

- Some fixed points repel the iterates; **these are called sources.**
- Other fixed points attract the iterates; **these are called sinks.**

Basically $f'(x) < 1$ are sinks and $f'(x) > 1$ are sources

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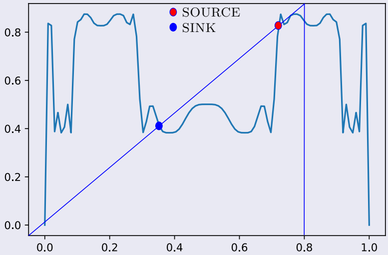
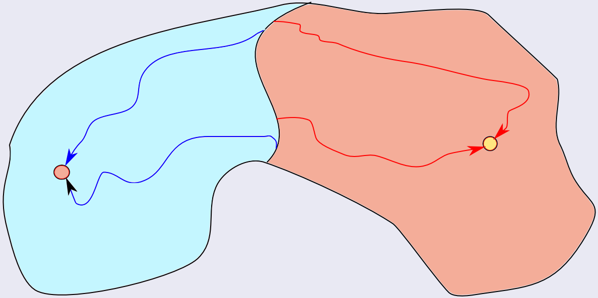


Areas of attraction

Basically, we have that there are areas the pull in the iterations of the function



These fixed points

In Deep Structures as RNN without sigmoid functions

$$
\begin{aligned}
\boldsymbol{h}_t &= W_{sd}\boldsymbol{x}_t + U_{ss}\boldsymbol{h}_{t-1} \\
\boldsymbol{y}_t &= V_{os}\boldsymbol{h}_t
\end{aligned}
$$

We have

$$
\boldsymbol{x}_t = V_{os}\left[W_{sd}\boldsymbol{x}_t + U_{ss}\boldsymbol{h}_{t-1}\right]
$$

Therefore if $\boldsymbol{b} = V_{os}U_{ss}\boldsymbol{h}_{t-1}$

- Then, we have that

$$
\boldsymbol{x}_t = V_{os}W_{sd}\boldsymbol{x}_t + V_{os}U_{ss}\boldsymbol{h}_{t-1} = I\boldsymbol{x}_t + 0
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\boldsymbol{x}_t = V_{os}W_{sd}\boldsymbol{x}_t + V_{os}U_{ss}\boldsymbol{h}_{t-1} = I\boldsymbol{x}_t + 0
$$

Therefore

We have that

$$V_{os}W_{sd} \approx I \text{ and } \boldsymbol{h}_{t-1} \approx 0$$

They define an area

Where V_{os} and W_{sd}

- They are the inverse of each other

And the hidden state is almost zero

- Basically they fixed point converts a RNN without activation functions in a linear model

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Gradient Clipping

We prevent gradient from blowing up by rescaling to a certain value

$$\|\nabla_\theta L\| > \eta \implies \nabla_\theta L = \frac{\eta \nabla_\theta L}{\|\nabla_\theta L\|}$$

We have a series of nice analysis [28]

$$\min_{x \in \mathbb{R}^d} f(x)$$

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$$S = \{x | \exists y \text{ such that } f(y) \leq f(x_o), \text{ and } \|x - y\| \leq 1\}$$

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We have then for S

In $\mathbb{R}^2$ the following example

Assumptions

Assumption 1

- Function f is lower bounded by f^*

Assumption 2

- Function f is twice differentiable

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Then, there are the following proposals

The ordinary gradient descent

$$x_{k+1} = x_k - \eta \nabla f(x_k)$$

The Clipped Gradient Descent (CGD)

$$x_{k+1} = x_k - h_c \nabla f(x_k), \text{ where } h_c = \min\left\{\eta_c, \frac{\gamma\eta_c}{\|\nabla f(x)\|}\right\}$$

Normalized Gradient Descent (NGD)

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Remark

Clipped GD and NGD are almost equivalent

- If we set $\gamma\eta_c = \eta_n$ and $\eta_c = \frac{\eta_n}{\beta}$ then

$$\frac{1}{2}h_c \leq h_n \leq 2h_c$$

A Natural Question

Definition

- The objective f is called L-smooth if $\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|$ for all $x, y \in \mathbb{R}^d$

This is equivalent under a twice differentiable f

$$\left\|\nabla^2 f(x)\right\| \leq L$$

Then, you get the following upper-bound

$$f(y) \approx f(x) + \nabla^T f(x)(y - x) + \frac{1}{2}(y - x)^T \nabla^2 f(x)(y - x)$$

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We have that

$$f(y) \leq f(x) + \nabla^T f(x)(y - x) + \frac{1}{2} L \|y - x\|^2$$

Then fixing all the other variables and assuming $y = x - h\nabla f(x)$

$$h^* = \arg\min_h \left[f(x) - h \|\nabla f(x)\|^2 + \frac{1}{2} L h^2 \|\nabla f(x)\|^2 \right] = \frac{1}{L}$$

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- "Is clipped gradient descent optimized for a different smoothness condition?"

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Then, we have

$$L(x) = \frac{\left\|\nabla f(x)\right\| + \beta}{\eta}$$

Assumption 3 by using $\left\|\nabla^2 f(x)\right\| \leq L$

- (L_0, L_1)-smoothness. f is (L_0, L_1)-smooth, if there exist positive L_0 and L_1 such that $\left\|\nabla^2 f(x)\right\| \leq L_0 + L_1 \left\|\nabla f(x)\right\|$
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The final Theorem

Theorem (CGD) [28]

- Assume that Assumptions 1, 2, and 3 hold in set S. With parameters

$$\eta_c = \frac{1}{10L_o} \text{ and } \gamma = \min\left\{\frac{1}{\eta_c}, \frac{1}{10L_o\eta_c}\right\},$$

  - Then Clipped GD terminates in

$$\frac{20L_0\left(f\left(x_0\right)-f^*\right)}{\epsilon^2}+\frac{20\max\left\{1,L_1^2\right\}\left(f\left(x_0\right)-f^*\right)}{L_0} \text{ iterations}$$

Remarks

The paper

- It points out to a high correlation between the Jacobian and the Hessian

There are more work to be done

- Please read the paper...

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Outline

Another way to stabilize the network

Data Normalization

- Standardization is the most popular form of preprocessing
  - Normally mean subtraction and subsequent scaling by the standard deviation.

Mean subtraction

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \text{ then } x_i^c = x_i - \mu$$

Finally

- Standardization refers to altering the data dimensions such that they are of approximately the same scale.

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Therefore, we have that

Standardization

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{x}_i - \mu)^2$$

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However, there other tricks, Bengio et al [29]

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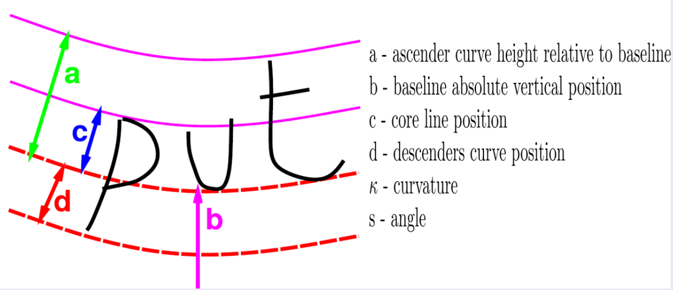

Softmax Scaling

Thus

- All new features have zero mean and unit variance.

Further

- Other linear techniques limit the feature values in the range of $[0, 1]$ or $[-1, 1]$ by proper scaling.

However

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Steps of Softmax Scaling

Softmax Scaling

- It consists of two steps

First one

$$y_{ik} = \frac{x_{ik} - \overline{x}_k}{\sigma} \quad (2)$$

Second one

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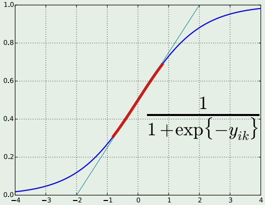
Explanation

Notice the red area is almost flat!!!



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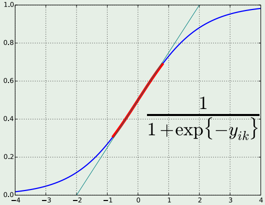

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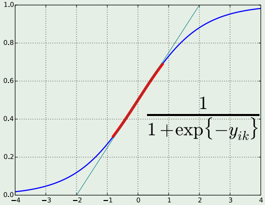

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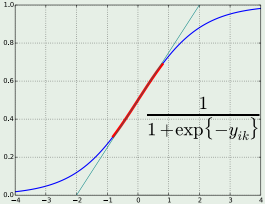

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Here, the people at Google [17] around 2015

They commented in the "Internal Covariate Shift Phenomena"

- Due to the change in the distribution of each layer's input

They claim

- The min-batch forces to have those changes which impact on the learning capabilities of the network.

In Neural Networks, they define this

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Consider a layer with the input u that adds the learned bias b

- Then, it normalizes the result by subtracting the mean of the activation over the training data:

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Now, if the gradient ignores the dependence of $E\left[x\right]$ on b

- Then $b = b + \Delta b$ where $\Delta b \propto -\frac{\partial l}{\partial \widehat{x}}$

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Normalization via Mini-Batch Statistic

It is possible to describe the normalization as a transformation layer

$$\widehat{\boldsymbol{x}} = Norm\left(\boldsymbol{x}, \mathcal{X}\right)$$

- Which depends on all the training samples $\mathcal{X}$ which also depends on the layer parameters

For back-propagation, we will need to generate the following terms

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Normalization via Mini-Batch Statistic

Problem!!!

- whitening the layer inputs is expensive, as it requires computing the covariance matrix

$$Cov\left[\boldsymbol{x}\right] = E_{\boldsymbol{x}\in\mathcal{X}}\left[\boldsymbol{x}\boldsymbol{x}^T\right] \text{ and } E\left[\boldsymbol{x}\right]E\left[\boldsymbol{x}\right]^T$$

  - To produce the whitened activations

Therefore

A Better Options, we can normalize each dimension

$$\widehat{\boldsymbol{x}}^{(k)} = \frac{\boldsymbol{x}^{(k)} - \mu}{\sigma}$$

- with $\mu = E\left[\boldsymbol{x}^{(k)}\right]$ and $\sigma^2 = Var\left[\boldsymbol{x}^{(k)}\right]$

This allows to speed up convergence

- Simply normalizing each input of a layer may change what the layer can represent.

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The Linear transformation

$$\boldsymbol{y}^{(k)} = \gamma^{(k)}\widehat{\boldsymbol{x}}^{(k)} + \beta^{(k)}$$

The parameters $\gamma^{(k)}, \beta^{(k)}$

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Batch Normalizing Transform

Input: Values of $\boldsymbol{x}$ over a mini-batch: $\mathcal{B} = \{\boldsymbol{x}_{1\ldots m}\}$, Parameters to be learned: γ, β

Output: $\{y_i = BN_{\gamma,\beta}(\boldsymbol{x}_i)\}$

1. $\mu_{\mathcal{B}} = \frac{1}{m}\sum_{i=1}^{m} x_i$
2. $\sigma_{\mathcal{B}}^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu)^2$
3. $\widehat{x} = \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$
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Finally

Batch Normalizing Transform

Input: Values of $\boldsymbol{x}$ over a mini-batch: $\mathcal{B} = \{\boldsymbol{x}_{1\ldots m}\}$, Parameters to be learned: γ, β

Output: $\{y_i = BN_{\gamma,\beta}(\boldsymbol{x}_i)\}$

1. $\mu_{\mathcal{B}} = \frac{1}{m}\sum_{i=1}^{m} \boldsymbol{x}_i$
2. $\sigma_{\mathcal{B}}^2 = \frac{1}{m}\sum_{i=1}^{m} (\boldsymbol{x}_i - \mu)^2$
3. $\widehat{\boldsymbol{x}} = \frac{\boldsymbol{x}_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$
4. $\boldsymbol{y}_i = \gamma^{(k)}\widehat{\boldsymbol{x}}_i + \beta = BN_{\gamma,\beta}(\boldsymbol{x}_i)$

Backpropagation

We have the following equations by using the loss function l

1. $\frac{\partial l}{\partial \widehat{\boldsymbol{x}}_i} = \frac{\partial l}{\partial \boldsymbol{y}_i} \times \gamma$
2. $\frac{\partial l}{\partial \sigma_{\mathcal{B}}^2} = \sum_{i=1}^{m} \frac{\partial l}{\partial \widehat{\boldsymbol{x}}_i} \times (\boldsymbol{x}_i - \mu_{\mathcal{B}}) \times \left(-\frac{1}{2}\right) \times \left(\sigma_{\mathcal{B}}^2 + \epsilon\right)^{-\frac{3}{2}}$
3. $\frac{\partial l}{\partial \mu_{\mathcal{B}}} = \left(\sum_{i=1}^{m} \frac{\partial l}{\partial \widehat{\boldsymbol{x}}_i} \times \frac{-1}{\sqrt{\sigma_{\mathcal{B}}^2+\epsilon}}\right) + \frac{\partial l}{\partial \sigma_{\mathcal{B}}^2} \times \frac{\sum_{i=1}^{m} -2\times(\boldsymbol{x}_i - \mu_{\mathcal{B}})}{m}$
4. $\frac{\partial l}{\partial \boldsymbol{x}_i} = \frac{\partial l}{\partial \widehat{\boldsymbol{x}}_i} \times \frac{1}{\sqrt{\sigma_{\mathcal{B}}^2+\epsilon}} + \frac{\partial l}{\partial \sigma_{\mathcal{B}}^2} \times \frac{2\times(\boldsymbol{x}_i - \mu_{\mathcal{B}})}{m} + \frac{\partial l}{\partial \mu_{\mathcal{B}}} \times \frac{1}{m}$
5. $\frac{\partial l}{\partial \gamma} = \sum_{i=1}^{m} \frac{\partial l}{\partial \boldsymbol{y}_i} \times \widehat{\boldsymbol{x}}_i$
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Input: Network N with trainable parameters Θ; subset of activations $\left\{ \boldsymbol{x}^{(k)} \right\}_{k=1}^{K}$

Output: Batch-normalized network for inference N_{BN}^{inf}

$N_{BN}^{tr} = N$ // Training BN network
for $k = 1 \ldots K$ do
    Add transformation $y^{(k)} = BN_{\gamma^{(k)}, \beta^{(k)}}\left(x^{(k)}\right)$ to N_{BN}^{tr}
    Modify each layer in N_{BN}^{tr} with input $x^{(k)}$ to take $y^{(k)}$instead
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    Process multiple training mini-batches $\mathcal{B}$, each of size m, and average over them
    $E[x] = E_{\mathcal{B}}[\mu_{\mathcal{B}}]$ and $Var[x] = \frac{m}{m-1}{}_{\mathcal{B}}\left[\sigma_{\mathcal{B}}^{2}\right]$
    In N_{BN}^{inf}, replace the transform $y = BN_{\gamma,\beta}(x)$ with

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Santurkar et al. [18]

- They found thats is not the covariance shift the one affected by it!!!

Santurkar et al. recognize that

- Batch normalization has been arguably one of the most successful architectural innovations in deep learning.

They used a standard Very deep convolutional network

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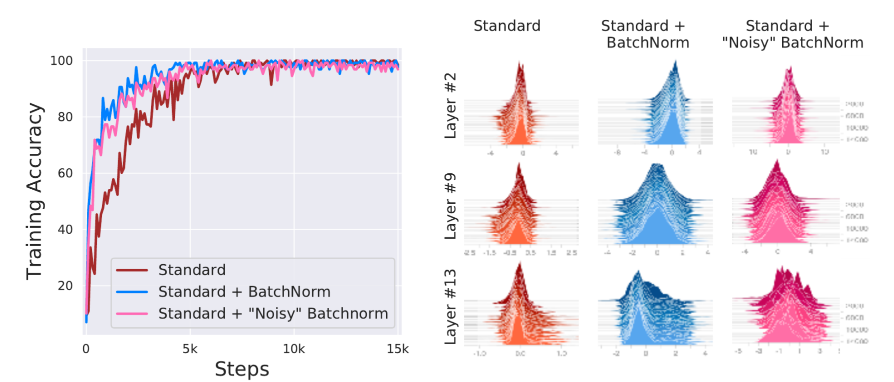
They found something quite interesting

The following facts



Actually Batch Normalization

It does not do anything to the Internal Covariate Shift

- Actually smooth the optimization manifold
  - It is not the only way to achieve it!!!

They suggest that

- "This suggests that the positive impact of BatchNorm on training might be somewhat serendipitous."

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They actually have a connected result

To the analysis of gradient clipping!!!

- They are the same group

Theorem (The effect of BatchNorm on the Lipschitzness of the loss)

- For a BatchNorm network with loss $\hat{\mathcal{L}}$ and an identical non-BN network with (identical) loss $\mathcal{L}$,

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They actually have a connected result

To the analysis of gradient clipping!!!

- They are the same group

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Outline

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Degradation Problem

- With the network depth increasing, accuracy gets saturated (which might be unsurprising) and then degrades rapidly.

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Therefore, we need to deal with such problems

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- He, Kaiming et al. - "Deep Residual Learning for Image Recognition"

Basically they got two layers doing something to an input

$$\mathcal{F}(\boldsymbol{x}) = A_2 A_1 \boldsymbol{x}$$

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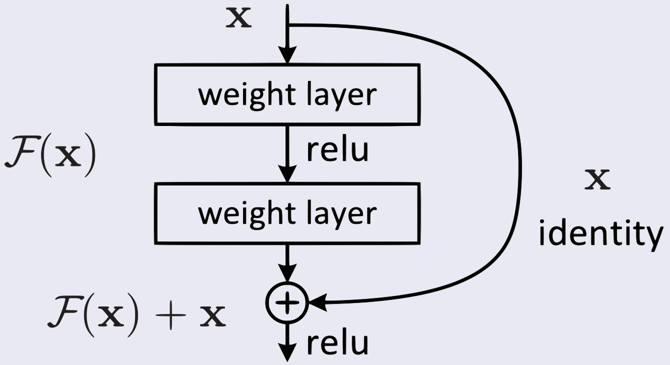
Blocks of the Original RNN

We have



A Winner

Something Notable

- Winner of ILSVRC 2015 in image classification, detection, and localization, as well as Winner of MS COCO 2015 detection, and segmentation.

Outline

We have seen many concepts

Deep Forward Networks

- Although a simple idea

They represent a rich field of study

- Basically... From Lower Complexity Features toward more complex more informative!!!

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