Introduction to Neural Networks and Deep Learning

Learning Process

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September 14, 2019

Outline

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Learning

How do we define learning in a Neural Network?

The property that is of primary significance for a neural network is

- The ability of the network to learn from its environment.
- To improve its performance through learning.

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"Learning is a process by which the free parameters of neural network are adapted through a process of stimulation by the environment in which the network is embedded. The type of learning is determined by the manner in which the parameter changes take place."

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Implications of the Definition of Learning

First

The neural network is stimulated by an environment.

Second

The neural network undergoes changes in its free parameters as a result of this stimulation.

Third

The neural network responds in a new way to the environment because of the changes that have occurred in its internal structure.

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Solution of Learning Problem $\approx$ Learning Algorithm

Quite Interesting

There is no unique learning algorithm for the design of neural networks.

What we have is more

A "kit of tools" represented by a diverse variety of learning algorithms:

- They depend on the type of architecture used in the Neural Network!!!

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Error Signal

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This is used

- As a control mechanism.

The strategy to be followed

- It is s to apply a sequence of corrective adjustments to the synaptic weights of neuron k.

What do we want?

We want

$$\lim_{t \to \infty} y_k(t) = d_k$$

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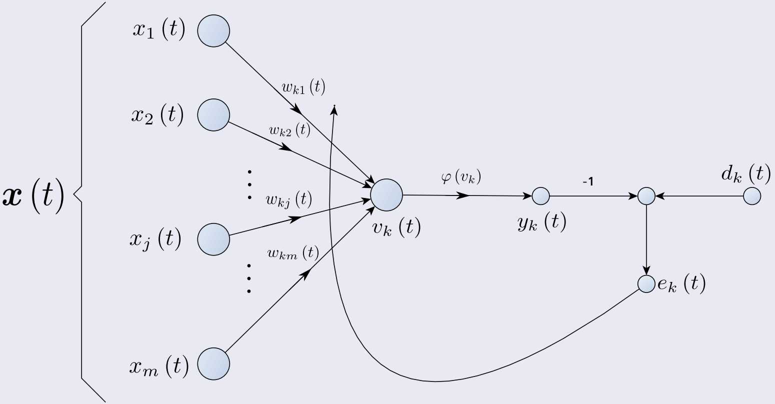
The Architecture

Graphical Representation



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We need to use this error in some way

For this we use a well know convex function

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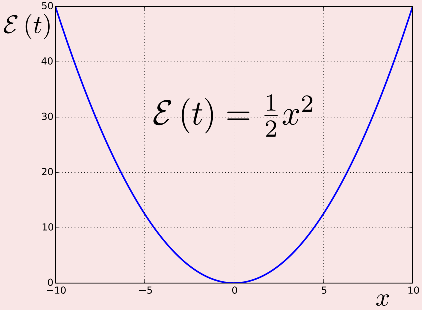

Error Correcting Learning

Cost Function with the error in it

$$\mathcal{E}(t) = \frac{1}{2} e_k^2(t) \tag{2}$$

We need to minimize this error

For this, we use the a learning rule called Delta Rule or Widrow-Hoff Rule [2]!!

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How this is done

We take derivative (Gradient) with respect to $w_{kj'}$ by imagining that we fix all the other values

$$\mathcal{E}(t) = \frac{1}{2}\left(d_k(t) - \sum_{j=0}^{m} w_{kj}(t)x_j(t)\right)^2$$

Assuming: $y_k(t) = \sum_{j=0}^{m} w_{kj}(t)x_j(t)$

Thus, we have

$$\mathcal{E}(t) = \frac{1}{2}\left(C_k(t) - w_{kj'}(t)x_{j'}(t)\right)^2$$

Where: $C_k(t) = d_k(t) - \sum_{\substack{j=1 \\ j \neq j'}}^{m} w_{kj}(t)x_j(t)$

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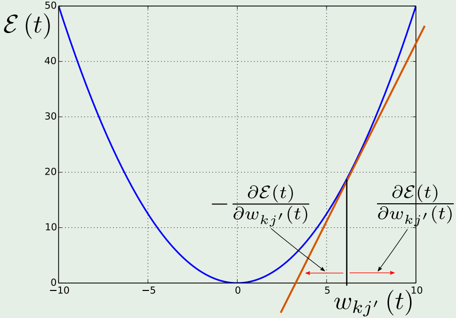
We have something like this

The Intuitive Idea



What information do we get from the Gradient?

We get the following information

The directions of greatest change in the axis of $w_{kj}(t)$

We use this to adjust the learning

For each weight element storing information for the neuron k.

Thus, we can state the delta rule [2]

"***The adjustment made to a synaptic weight of a neuron is proportional to the product of the error signal and the input signal of the synapse in question.***"

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This is simple

It comes from taking the gradient

$$\frac{\partial \mathcal{E}(t)}{\partial w_{kj'}} = \frac{\partial \frac{1}{2}\left(C_k(t) - w_{kj'}(t)x_{j'}(t)\right)^2}{\partial w_{kj'}}$$
$$= -\left(C_k(t) - w_{kj'}(t)x_{j'}(t)\right)x_{j'}(t)$$
$$= -e_k(t)\,x_{j'}(t)$$

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Thus, we get

Delta Rule or Widrow-Hoff Rule

$$\Delta w_{kj'}(t) = \eta e_k(t)\, x_{j'}(t) \tag{3}$$

With η absorbing the negative sign and representing the learning rate!!!

Actually this is know as Gradient Descent

$$w_{kj'}(t+1) = w_{kj'}(t) + \Delta w_{kj'}(t) \tag{4}$$

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In effect, $w_{kj'}(t)$ and $w_{kj'}(t+1)$ may be viewed as the ***old*** and ***new*** values of synaptic weight $w_{kj'}$, respectively.

In computational method, we can also write that

$$w_{kj'}(t) = z^{-1}\left[w_{kj'}(t+1)\right],$$

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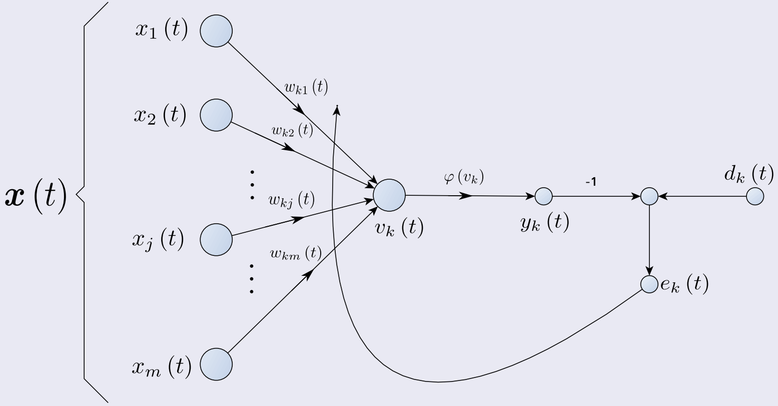
The Error-Correction Architecture

We have the graphical representation of the error-correction learning



This is an example of a closed-loop feedback system

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What is this?

We store information

All (or most) of the past experiences are explicitly stored in a large memory of correctly classified input-output examples.

Formally

Input-Output examples : $\{(x_i, d_i)\}_{i=1}^{N}$

Where

- x_i denotes an input vector.
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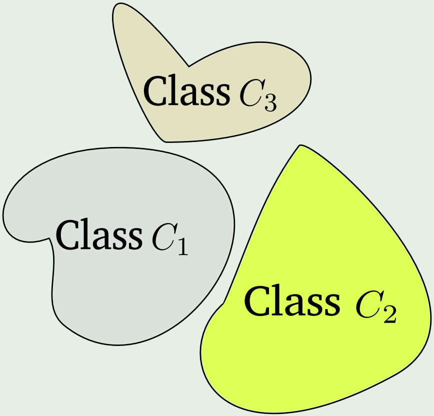
In addition, we split these samples into classes

Each of these inputs are in a Class C_j



Then

Given all this information

We can build an algorithm to classify not seen before samples $\boldsymbol{x}_{test}$.

This algorithm works as follow

The algorithm responds by retrieving and analyzing the training data in a "local neighborhood" of $\boldsymbol{x}_{test}$.

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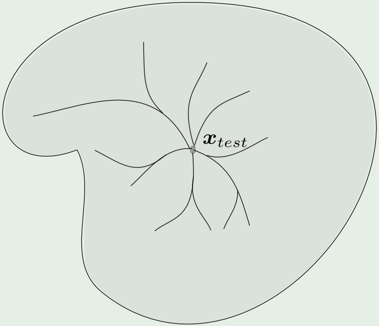
Ingredients

First

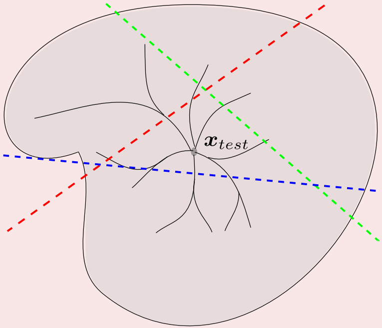
Criterion used for defining the local neighborhood of the test vector $\boldsymbol{x}_{test}$.



Ingredients

Second

Learning rule applied to the training examples in the local neighborhood of $\boldsymbol{x}_{test}$.



Remark

Something Notable

Algorithms are different between each other depending on how these two ingredients are defined

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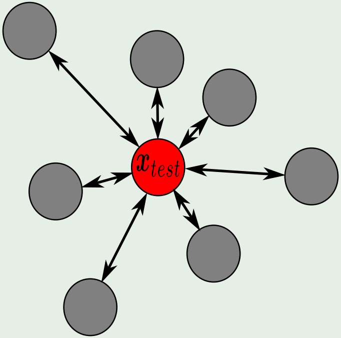
Example: K-Nearest Neighbor Classifier

First Step

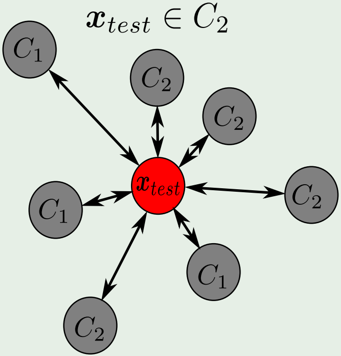
Identify the k classified patterns that lie nearest to the test vector $\boldsymbol{x}_{test}$ for some integer k.



Example: K-Nearest Neighbor Classifier

Second Step

Assign $\boldsymbol{x}_{test}$ to the class that is most frequently represented in the k nearest neighbors to $\boldsymbol{x}_{test}$.



Cover and Hart (1967) have studied the nearest neighborhood rule (Simplest Version of K-NN)

Their analysis (Chapter 3) is based in the following assumptions

- The classified examples $(\boldsymbol{x}_i, d)$ are independently and identically distributed (iid), according to the joint probability distribution of the example $(\boldsymbol{x}, d)$.
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Hebb's postulate of learning

It is the oldest of all learning rules

In Hebb's "The Organization of Behavior (1949, p.62)"[3]:

- When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic changes take place in one or both cells such that A's efficiency as one of the cells firing B, is increased.

This can be rephrased as...

Something Notable

It is possible to expand this as a two-part rule...(Stent. 1973; Changeux and Danchin. 1976)

First

If two neurons on either side of a synapse (connection) are activated simultaneously (i.e., synchronously), then the strength of that synapse is selectively increased.

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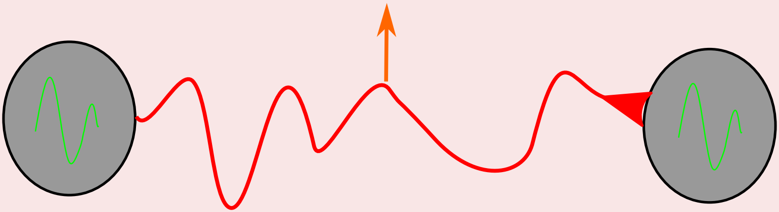

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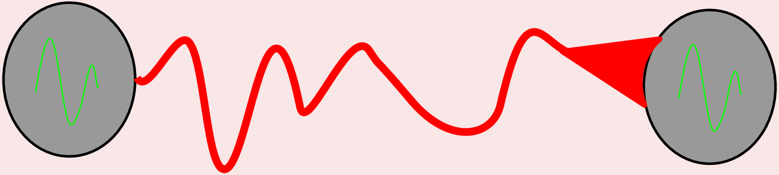

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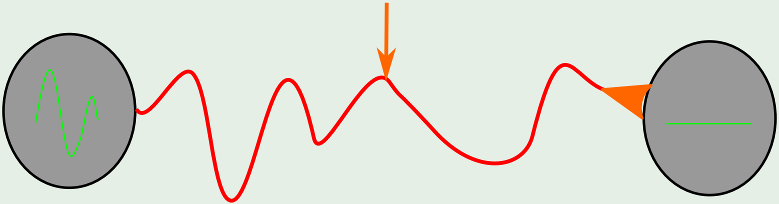
If two neurons on either side of a synapse are activated asynchronously, then that synapse is selectively weakened or eliminated.



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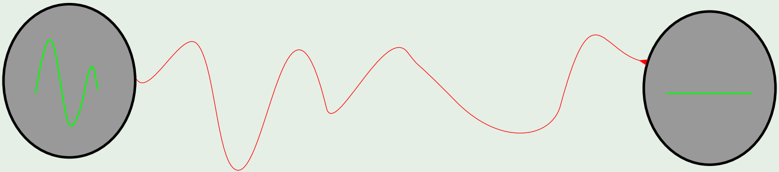

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The Hebbian Synapse

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A Hebbian synapse is a synapse that uses a:

1. Time-Dependant
2. Highly Local
3. Strongly Interactive

Mechanism to increase synaptic efficiency as a function of the correlation between the presynaptic and postsynaptic activities.

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Key Mechanism of Hebbian Synapse

Time-dependent mechanism

It refers to the dependence of the synapse to the exact time of occurrence of the presynaptic and postsynaptic signals.

Local Mechanism

It refers to how the local information is used by the Hebbian synapse to make modifications to the synapse itself.

Interactive Mechanism

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Conjunctional or correlational mechanism

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Where

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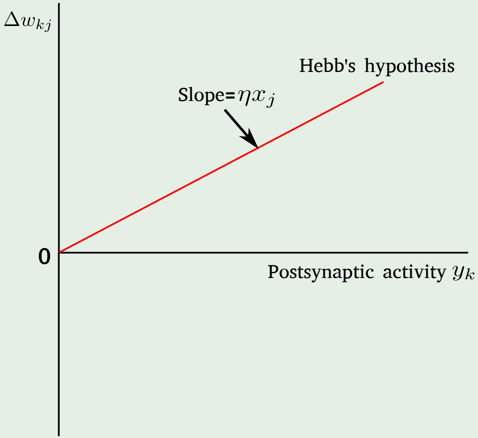
Where

- η is a positive constant that determines the learning rate.
- Clearly (Eq. 5) emphasizes the correlational nature of a Hebbian rate of synapse.
- It is referred as the **activity product rule**.

Graphical Representation

Δw_{kj} plotted versus the output signal (Quite similar to RELU)



Problem

Something Notable

The repeated application of the input signal (presynaptic activity) x_j leads to an increase in y_k

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It leads to exponential growth that finally drives the synaptic connection into saturation!!!

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To solve this, we have...

Covariance Hypothesis (Sejnowski, 1974) [4]

In this hypothesis, the presynaptic and postsynaptic signals are replaced by the departure of presynaptic and postsynaptic signals from their respective average values over a certain time interval.Xi

Given the values $\overline{x}$ and $\overline{y}$ with respect to time

$$\Delta w_{kj} = \eta \left(x_j - \overline{x} \right) \left(y_k - \overline{y} \right) \quad (6)$$

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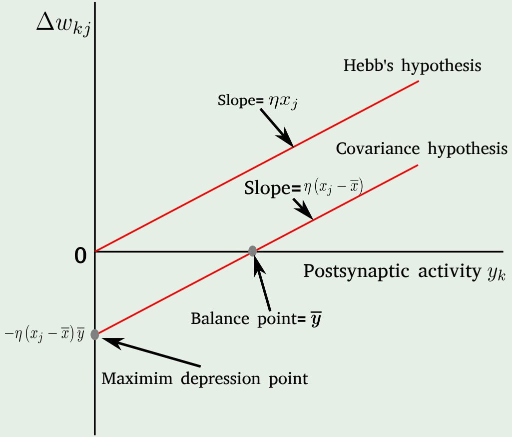

Properties of the Covariance Hypothesis

The covariance hypothesis allows for the following

- Convergence to a nontrivial state, which is reached when $x_k = \overline{x}$ and $y_j = \overline{y}$.
- Prediction of both synaptic potentiation (i.e., increase in synaptic strength) and synaptic depression (i.e., decrease in synaptic strength).

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This behavior may be regarded as a form of temporal competition between the incoming patterns.

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Outline

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The meaning

Intuition

In competitive learning the output neurons of a neural network compete among themselves to become active (fired).

Not only that

In Hebbian learning several output neurons may be active simultaneously, but in competitive learning only a single output neuron is active at any one time!!!

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This makes competitive learning highly suited to discover statistically salient features that may be used to classify a set of input patterns.

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Basic Elements (Rumelhart and Zipser, 1985) [5]

First

A set of neurons that are all the same except for some randomly distributed synaptic weights, and which therefore respond differently to a given set of input patterns.

Second

A limit imposed on the "strength" of each neuron.

- The neuron that wins the competition is called a winner-take-all neuron.

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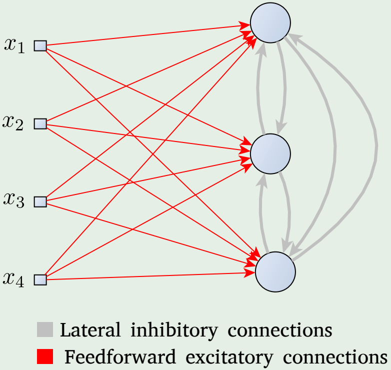
Simple Example

Architecture of a competitive learning network



Mathematical Model

We want the following

For a neuron k to be the winning neuron, its induced local field v_k, for a specified input pattern x must be the largest among all the neurons in the network.

Thus, we have

$$y_k = \begin{cases} 1 & \text{if } v_k > v_j \ \forall j, j \neq k \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

With

v_k represents the combined action of all the forward and feedback inputs to neuron k.

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In addition

We want each neuron to have a fixed amount of synaptic weight

$$\sum_j w_{kj} = 1 \quad (8)$$

Thus

A neuron learns by shifting synaptic weights from inactive to active inputs.

Then, we have the following competitive learning rule

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Then

What is going to happened?

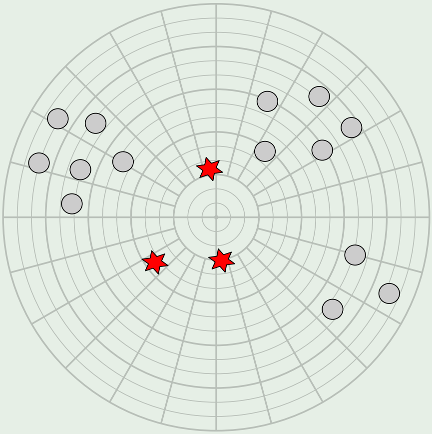
$$\boldsymbol{w}_k \longrightarrow \boldsymbol{x} \tag{10}$$

i.e. moving the weight in neuron k toward the pattern $\boldsymbol{x}$.

Example

We have then (Given $\sum_j w_{kj} = 1$)

Here, the gray dots are the patterns and the red stars are the state of the networks!!!



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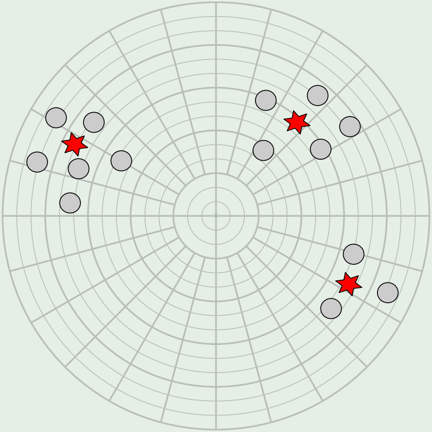

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The ability of a neural network to perform **clustering** through competitive learning.

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Outline

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Boltzmann Learning

Ludwig Boltzmann

- Ludwig Eduard Boltzmann was an Austrian physicist and philosopher (1844-1906).
- Greatest achievement was in the development of statistical mechanics, which explains and predicts how the properties of atoms determine the physical properties of matter.

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$$E = -\frac{1}{2}\sum_{\substack{j \\ j\neq k}}\sum_{k} w_{kj}x_k x_j \tag{11}$$

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Operation of a Boltzmann Machine

Take a neuron randomly

Then change the state of the neuron k from state x_k to state $-x_k$ at some temperature T with probability

$$P\left(x_k \longrightarrow -x_k\right) = \frac{1}{1 + \exp\left\{-\frac{\Delta E_k}{T}\right\}} \tag{12}$$

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Architecture for the Boltzmann Machines

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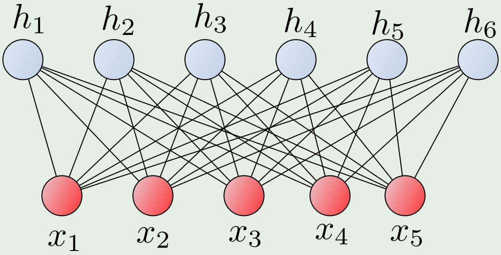
The neurons of a Boltzmann machine partition into two functional groups: visible and hidden.



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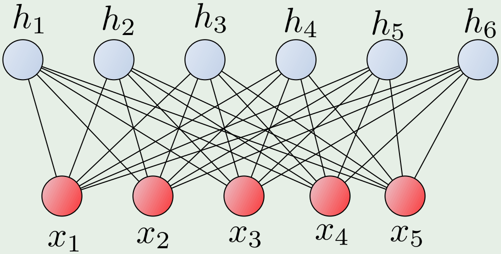

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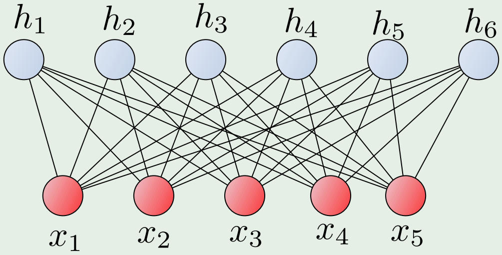

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Modes of Operation

Clamped condition

The visible neurons are all clamped onto specific states determined by the environment.

Free-running condition

All the neurons (visible and hidden) are allowed to operate freely.

- ρ_{kj} denotes the correlation between the states of neurons j and k under free running.

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A little taste

Here is one definition

$$
\begin{aligned}
\rho_{kj}^{+} &= \langle x_k x_j \rangle^{+} \\
&= \sum_{\boldsymbol{x}_\alpha \in T} \sum_{\boldsymbol{x}_\beta} P\left(\boldsymbol{X}_\beta = \boldsymbol{x}_\beta | \boldsymbol{X}_\alpha = \boldsymbol{x}_\alpha\right) x_k x_j
\end{aligned}
$$

- $\boldsymbol{x}_\alpha$ training inputs from environment.
- $\boldsymbol{x}_\beta$ hidden responses.

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Important

Both correlations are averaged over all possible states of the machine when it is in thermal equilibrium.

Boltzmann learning rule

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Chapter 11 Haykin's Book.

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Outline

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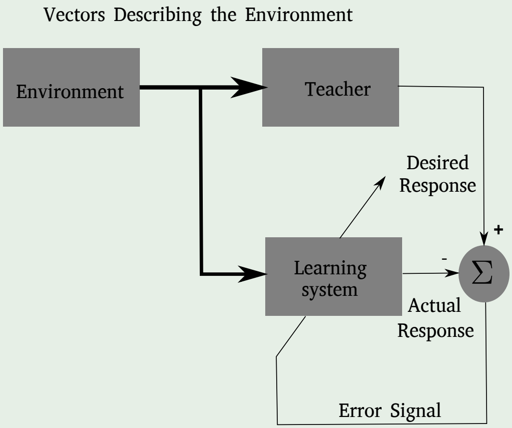
A Graphical View

Error Correcting Version



Outline

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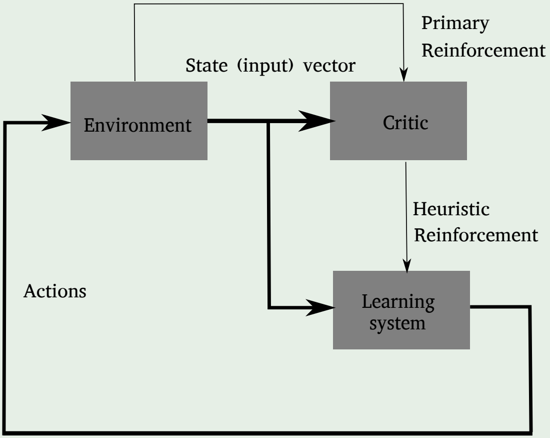
Reinforcement learning/Neurodynamic programming

Barto et al., 1983



Remark: Reinforcement learning is closely related to dynamic programming, which was developed by Bellman (1957) in the context of optimal control theory.

Observations on Reinforcement Learning

First

We want to minimize a cost-to-go function defined as the expectation of the cumulative cost of actions taken over a sequence of steps.

Second

The system is designed to learn under delayed reinforcement.

Delayed Reinforcement

The system observes a temporal sequence of stimuli (i.e., state vectors) also received from the environment, which eventually result in the generation of the heuristic reinforcement signal.

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It is difficult to perform

- There is no teacher to provide a desired response at each step of the learning process.
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Unsupervised Learning

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In unsupervised or self-organized learning there is no external teacher or critic to oversee the learning process

Something Notable

Provision is made for a taskindependent measure of the quality of representation that the network is required to learn, and the free parameters of the network are optimized with respect to that measure

How is this done?

This develops the ability to form internal representations for encoding features of the input and thereby to create new classes automatically (Becker, 1991).

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Learning Tasks

Pattern Association

- An associative memory is a brainlike distributed memory that learns by association.

$$\boldsymbol{x_k} \underset{\boldsymbol{Associate}}{\longrightarrow} \boldsymbol{y}_k, k = 1, 2, ..., q$$

Learning Tasks

Pattern Recognition

- It is is formally defined as the process whereby a received pattern/signal is assigned to one ofa prescribed number of classes (categories).

Learning Tasks

Function Approximation

- Consider an input-output mapping $d = f(\boldsymbol{x})$
- The requirement is to design a neural network that approximates the unknown function $f(\boldsymbol{x})$ using a function $F(\boldsymbol{x})$

$$\|F(\boldsymbol{x}) - f(\boldsymbol{x})\| < \epsilon, \forall \boldsymbol{x}$$

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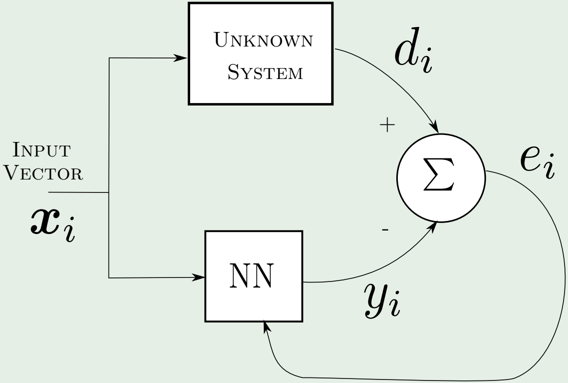
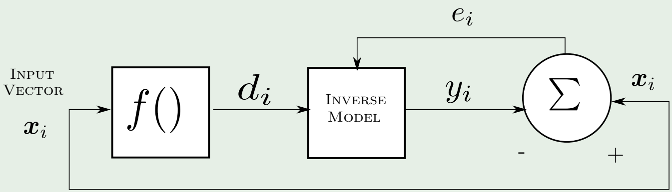
Function approximation can be used

System Identification



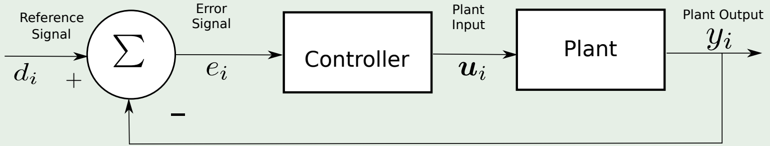
Function approximation can be used

Inverse system



In addition

Neural Networks can be used for Control



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Finally, Filtering

We may use a filter to perform three basic information processing tasks

1. **Filtering.** This task refers to the extraction of information about a quantity of interest at discrete time n by using data measured up to and including time n.
2. **Smoothing.** This second task differs from filtering in that information about the quantity of interest need not be available at time n, and data measured later than time n can be used in obtaining this information.
3. **Prediction.** This task is the forecasting side of information processing.

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