# Introduction to Neural Networks and Deep Learning Introduction to Neural Networks

Andres Mendez-Vazquez

September 9, 2019

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# Outline

#### What are Neural Networks?

- Introduction
- Structure of a Neural Cell
- Pigeon Experiment
- Formal Definition of Artificial Neural Network
- Basic Elements of an Artificial Neuron
  - A Simple Example
  - A More Complex Example
- Types of Activation Functions
  - McCulloch-Pitts model
  - More Advanced Models
- The Problem of the Vanishing Gradient
  - Fixing the Problem, ReLu function

#### 2 Neural Network As a Graph

- Introduction
- Feedback
- Neural Architectures
  - Single-Layer Feedforward Networks
  - Multilayer Feedforward Networks
  - Recurrent Networks
  - Deep Learning Architectures
- Knowledge Representation
- Design of a Neural Network
- Representing Knowledge in a Neural Networks



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# What are Neural Networks? [1]

### **Basic Intuition**

The human brain is a highly complex, nonlinear and parallel computer

#### It is organized as a

Network with (Ramon y Cajal 1911)



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- **2** Connections  $\approx$  Axons and Dendrites



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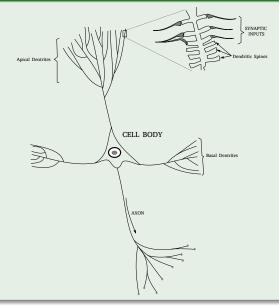
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# Example

## The Neural Structure



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# Pigeon Experiment

### Watanabe et al. 1995 [2]

Pigeons as art experts

#### Experiment

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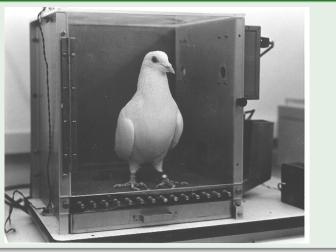
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## The Pigeon in the Skinner Box

## Something like this



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- Pigeons were able to discriminate between Van Gogh and Chagall with 95% accuracy (when presented with pictures they had been trained on).
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- They can extract and recognize patterns (the 'style')
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# Formal Definition [1]

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## Inter-neuron connection strengths?

### How do the neuron collect this information?

Some way to aggregate information needs to be devised...

#### A Classic

Use a summation of product of weights by inputs!!

#### Something like

$$\sum_{i=1}^m w_i \times x_i$$

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Where:  $w_i$  is the strength given to signal  $x_i$ 

However: We still need a way to regulate this "aggregation" (Activation function)

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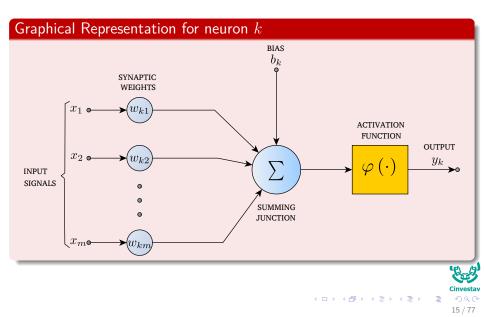
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# Basic Elements of an Artificial Neuron (AN) Model

### Set of Connecting links

- A signal  $x_j$ , at the input of synapse j connected to neuron k is multiplied by the synaptic weight  $w_{kj}$ .
- The weight may lie in a negative or positive range.
  - What about the real neuron? In classic literature you only have positive values.



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- It maps the permissible range of the output signal to an interval.

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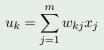
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### Adder



•  $x_1, x_2, ..., x_m$  are the input signals.

 $\bigcirc$   $w_{k1}, w_{k2}, ..., w_{km}$  are the synaptic weights.

It is also known as "Affine Transformation."

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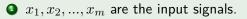
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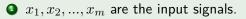
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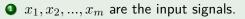
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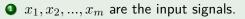
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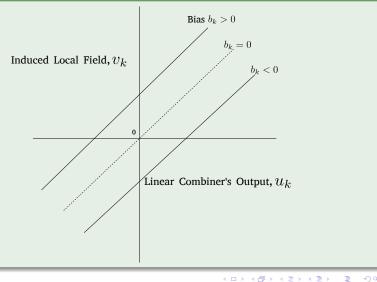
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## Integrating the Bias





Thus

## Final Equation

$$v_k = \sum_{j=0}^m w_{kj} x_j$$
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With

 $x_0 = 1$  and  $w_{k0} = b_k$ 



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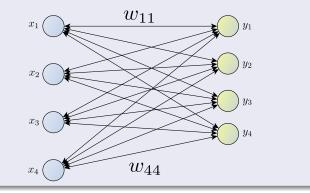
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## Energy Based Network

## Bidirectional Associative Memory (BAM) [3]





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# A Little of Linear Algebra

Here, we can denote the weights as  $n \times k$  matrix  ${oldsymbol W}$ 

- The n corresponds to the n dimensional vector  $oldsymbol{x}_0$
- The k corresponds to the k dimensional vector  $oldsymbol{y}_0$

Therefore the mapping is build in the following way given the feedback

 $\begin{aligned} & \boldsymbol{y}_0 = \operatorname{sgn}\left(\boldsymbol{x}_0\boldsymbol{W}\right) \\ & \boldsymbol{x}_1^T = \operatorname{sgn}\left(\boldsymbol{W}\boldsymbol{y}_0\right) \\ & \boldsymbol{y}_1 = \operatorname{sgn}\left(\boldsymbol{x}_1\boldsymbol{W}\right) \end{aligned}$ 



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## This is done until a stable state is reached

### Meaning

$$oldsymbol{y} = {
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#### A Notable Example

• The Hopfield Networks



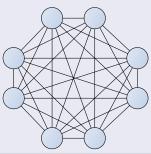
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# Types of Activation Functions I

## Threshold Function

$$\varphi(v) = \begin{cases} 1 & \text{if } v \ge 0\\ 0 & \text{if } v < 0 \end{cases} \text{ (Heaviside Function)} \tag{3}$$

the following picture

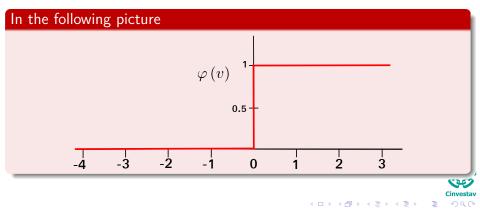


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(3)



#### We can use this activation function

To generate the first Neural Network Model

#### Clearly

The model uses the summation as aggregation operator and a threshold function.



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# McCulloch-Pitts model [4]

## McCulloch-Pitts model (Pioneers of Neural Networks in the 1940's)

Output

$$\label{eq:output_state} \mathsf{Output}\ y_k = \begin{cases} 1 & \text{ if } v_k \geq \theta \\ 0 & \text{ if } v_k < \theta \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if } v_k \le \theta = 0\\ 0 & \text{if } v_k > \theta = 0 \end{cases}$$

#### with induced local field $oldsymbol{w}_k = (1,1)^*$

$$v_k = \sum_{j=1}^m w_{kj} x_j + b_k$$



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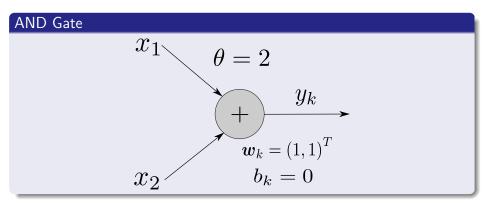
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(5)

(4)

It is possible to do classic operations in Boolean Algebra

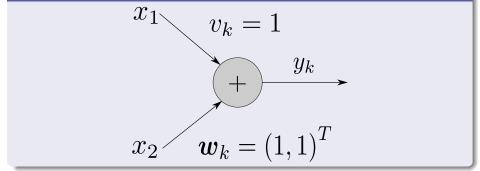




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## In the other hand

## OR Gate





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Finally

## NOT Gate



### Claude Shannon

• "A Symbolic Analysis of Relay and Switching Circuits"

 Shannon proved that his switching circuits could be used to simplify the arrangement of the electromechanical relays
 These circuits could solve all problems that Boolean algebra could solve.



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## More advanced activation function

#### **Piecewise-Linear Function**

$$\varphi(v) = \begin{cases} 1 & \text{if } v_k \ge \frac{1}{2} \\ v & \text{if } -\frac{1}{2} < v_k < \frac{1}{2} \\ 0 & \text{if } v \le -\frac{1}{2} \end{cases}$$

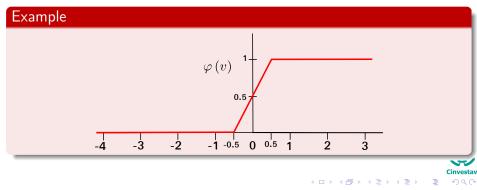
Example



(6)

### More advanced activation function

#### **Piecewise-Linear Function**



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### Remarks

#### Notes about Piecewise-Linear function

The amplification factor inside the linear region of operation is assumed to be unity.



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#### Special Cases

• A linear combiner arises if the linear region of operation is maintained without running into saturation.

The piecewise-linear function reduces to a threshold function if the

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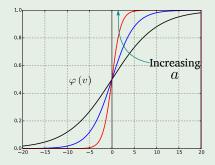
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## A better choice!!!

### Sigmoid function

$$\varphi\left(v\right) = \frac{1}{1 + \exp\left\{-av\right\}}$$

### Where a is a slope parameter.



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# The Problem of the Vanishing Gradient

### When using a non-linearity

• However, there is a drawback when using Back-Propagation (As we saw in Machine Learning) under a sigmoid function

$$s\left(x\right) = \frac{1}{1 + e^{-x}}$$

Because if we imagine a Deep Neural Network as a series of layer functions *[*,

$$y(A) = f_t \circ f_{t-1} \circ \cdots \circ f_2 \circ f_1(A)$$

• With  $f_t$  is the last layer.

Therefore, we finish with a sequence of derivatives  $\frac{\partial y(A)}{\partial w_{1i}} = \frac{\partial f_t(f_{t-1})}{\partial f_{t-1}} \cdot \frac{\partial f_{t-1}(f_{t-2})}{\partial f_{t-2}} \cdot \dots \cdot \frac{\partial f_2(f_1)}{\partial f_2} \cdot \frac{\partial f_1(A)}{\partial w_{1i}}$ 

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$\partial w_{1i} \qquad \partial f_{t-1}$	$\partial f_{t-2}$	$\partial f_2 \qquad \partial w_{1i}$	

#### Given the commutativity of the product

• You could put together the derivative of the sigmoid's

$$f(x) = \frac{ds(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$

#### Therefore, deriving again

$$\frac{df(x)}{dx} = -\frac{e^{-x}}{(1+e^{-x})^2} + \frac{2(e^{-x})^2}{(1+e^{-x})^3}$$

After making  $\frac{df(x)}{dx} = 0$ 

• We have the maximum is at x = 0



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After making 
$$\frac{df(x)}{dx} = 0$$

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### The maximum for the derivative of the sigmoid

• f(0) = 0.25

#### Therefore, Given a Deep Convolutional Network

We could finish with

$$\lim_{k \to \infty} \left( \frac{ds\left(x\right)}{dx} \right)^k = \lim_{k \to \infty} \left( 0.25 \right)^k \to 0$$

#### A Vanishing Derivative or Vanishing Gradient

 Making quite difficult to do train a deeper network using this activation function for Deep Learning and even in Shallow Learning



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## Thus

#### The need to introduce a new function

$$f\left(x\right) = x^{+} = \max\left(0, x\right)$$

#### is called ReLu or Rectifier

With a smooth approximation (Softplus function)

$$f\left(x\right) = \frac{\ln\left(1 + e^{kx}\right)}{k}$$

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### We have

### When k = 1

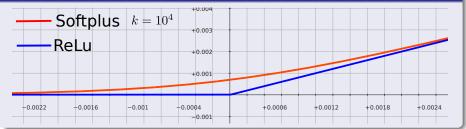




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Increase k

### When $k=10^4$





### **Final Remarks**

### Although, ReLu functions

#### • They can handle the problem of vanishing problem

# However, as we will see, saturation starts to appear as a problem

As in Hebbian Learning!!!



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#### Definition

A neural network is a directed graph consisting of nodes with interconnecting synaptic and activation links.



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### Properties

Each neuron is represented by an function

Each link represent a weight.

The weighted sum of the input signals defines the local field.

The activation function maps local field to an output



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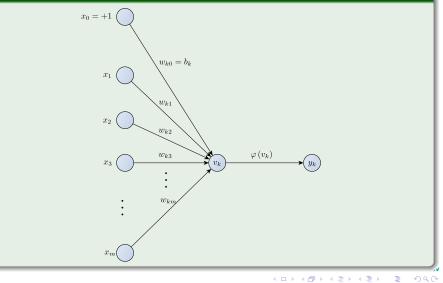
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Example

### Simple Perceptron



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### Observation

- A partially complete graph describing a neural architecture has the following characteristics:
  - Source nodes supply input signals to the graph.
  - Each neuron is represented by a single node called a computation node
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- Block diagram, providing a functional description of the network.
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  - Then one we plan to use.
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# Feedback Model

## Feedback

- Feedback is said to exist in a dynamic system.
  - Indeed, feedback occurs in almost every part of the nervous system of every animal (Freeman, 1975)

#### Feedback Architecture

#### Thus

Given  $x_i'$  the internal signal and A (The weights product) operator:

Output  $y_{k}\left(n
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ight)
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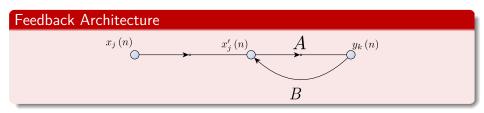


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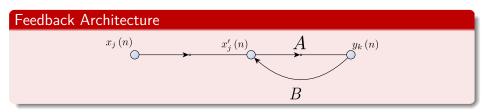


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Given  $x'_i$  the internal signal and A (The weights product) operator:

Output 
$$y_k(n) = A\left(x'_j(n)\right)$$
 (8)

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# Thus

## The internal input has a feedback

Given a  ${\cal B}$  operator:

$$x'_{j}(n) = x_{j}(n) + B(y_{k}(n))$$
 (9)

#### Then eliminating $x_i^\prime$ ( $x_i$

$$y_{k}\left(n\right) = \frac{A}{1 - AB}\left(x_{j}\left(n\right)\right) \tag{10}$$

Where <sup>A</sup>/<sub>1-AB</sub> is known as the closed-loop operator of the system.



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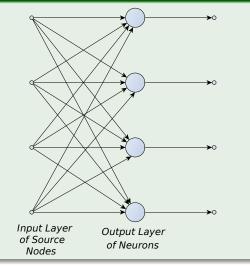
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# Single-Layer Feedforward Networks

## We begin with something quite simple



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## Observations

### Observations

This network is know as a strictly feed-forward or acyclic type.



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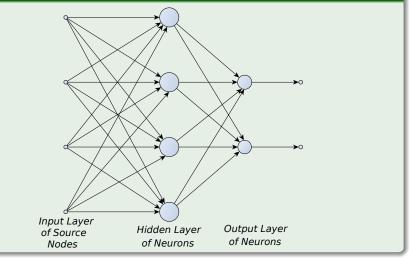
#### Multilayer Feedforward Networks

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## Multilayer Feedforward Networks

## Stacking Layers



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## Observations

### Observations

• This network contains a series of hidden layer.

Each hidden layers allows for classification of the new output space of the previous hidden layer.



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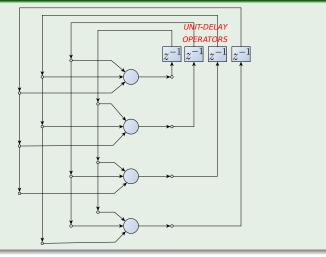
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## **Recurrent Networks**

## Connecting the back with the front



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## Observations

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This network has not self-feedback loops.

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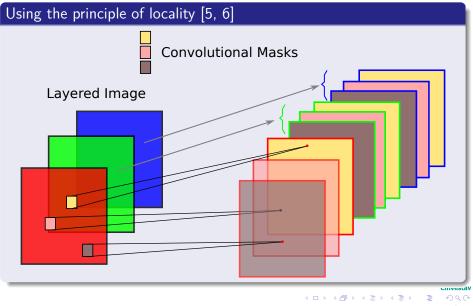
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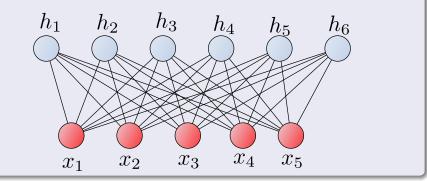
# Convolutional Deep Learners



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## Restricted Boltzmann Machines

## Energy Based Architectures [7]





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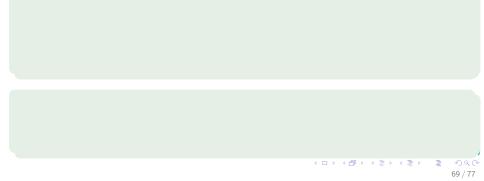
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### Thus, we have the following phases of designing a Neuronal Network

- O Choose appropriate architecture
- Train the network learning.
  - Use the Training Data!!!
- Test the network with data not seen before
  - Use a set of pairs that where not shown to the network so the y component is guessed.
- Then, you can see how well the network behaves Generalization Phase

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## Representing Knowledge in a Neural Networks

## Notice the Following

The subject of knowledge representation inside an artificial network is very complicated.

#### However: Pattern Classifiers Vs Neural Networks

 Pattern Classifiers are first designed and then validated by the environment.



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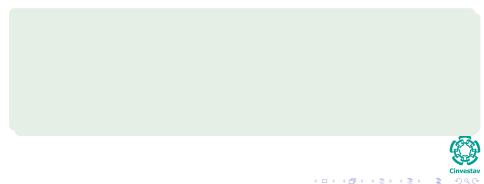
### Kurt Hornik et al. proved (1989)

"Standard multilayer feedforward networks with as few as one hidden layer using arbitrary squashing functions are capable of approximating any **Borel measurable function**" (Basically many of the known ones!!!)

# Rules Knowledge Representation

## Rule 1

- Similar inputs from similar classes should usually produce similar representation.
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$$d(x_i, x_j) = ||x_i - x_j||$$
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1 
$$d(\mathbf{x}_i, \mathbf{x}_j) = ||\mathbf{x}_i - \mathbf{x}_j||$$
 (Classic Euclidean Metric).  
2  $d_{ij}^2 = (\mathbf{x}_i - \boldsymbol{\mu}_i)^T \sum^{-1} (\mathbf{x}_j - \boldsymbol{\mu}_j)$  (Mahalanobis distance) where  
3  $\boldsymbol{\mu}_i = E[\mathbf{x}_i]$ .  
4  $\sum E[(\mathbf{x}_i - \boldsymbol{\mu}_i) (\mathbf{x}_i - \boldsymbol{\mu}_i)^T] = E[(\mathbf{x}_j - \boldsymbol{\mu}_j) (\mathbf{x}_j - \boldsymbol{\mu}_j)^T]$ .



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## More

#### Rule 2

• Items to be categorized as separate classes should be given widely different representations in the network.

#### Rule 3

• If a particular feature is important, then there should be a large number of neurons involved in the representation.

#### Rule 4

- Prior information and invariance should be built into the design:
  - Thus, simplify the network by not learning that data.



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