Machine Learning for Data Mining

Frequent Itemset Mining & Association Rules

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Outline

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Association Rule Discovery

In the Market-basket model

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The Market-Basket Model

A large set of items

For example, things sold in a supermarket.

A large set of baskets, which is a small subset of items

For example, the things one customer buys on one day.

In general, we have a many to many mapping (association) between two types of things

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Association Rules

Given a set of baskets

ID	Items
1	Bread,Coke,Milk
2	Beer,Bread
3	Beer,Coke,Diaper,Milk
4	Beer,Bread,Diaper,Milk
5	Coke,Diaper,Milk

We want to discover association rules

- People who bought $\{x, y, z\}$ tend to buy $\{v, w\}$.

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Itemsets

Basically

Given the baskets we want to find if an itemset (**Set of items**) is a likely set.

Association Rules

And given that

We want to generate likely association rules

Output:

Rules Discovered
{Milk}$\Rightarrow${Coke}
{Diaper,Milk}$\Rightarrow${Beer}

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Applications: Market Analysis

Items and Baskets

- Items are products at the store.
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Real market baskets

Chain stores keep Tera-bytes of data about what customers buy together

- It tells them how customers navigate stores, thus allowing them position tempting items

It suggests "marketing tricks", for example, run sales on diapers and raise the price of beer

- Nevertheless, This needs High Support (A lot of Data), or no Money!!!

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Problem

This is a complete NP-complete problem.

We avoid this problem by using the following trick

Given a graph

- Divide the nodes into two equal groups at random.

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Baskets = Nodes in the Left and Items = Nodes in the Right

The problem becomes on a search of complete bipartite subgraphs $K_{s,t}$ on a Bipartite Graph

- Thus, given a community kernel representing it, we add nodes from either of the two groups.

By Using a Simple Rule

- if those nodes have edges to many of the nodes already identified as belonging to the community.

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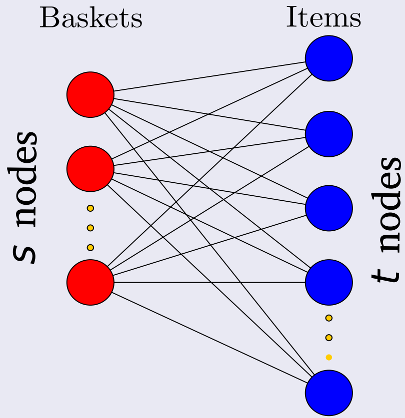
For Example



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How?

The members of the basket, for node v, are the nodes of the left side to which v is connected.

Let the support threshold be s

The number of nodes that the instance of $K_{s,t}$ has on the right side.

Looking for $K_{s,t}$ is like looking for a set of support s with a layer t

Or, all frequent itemsets of size t.

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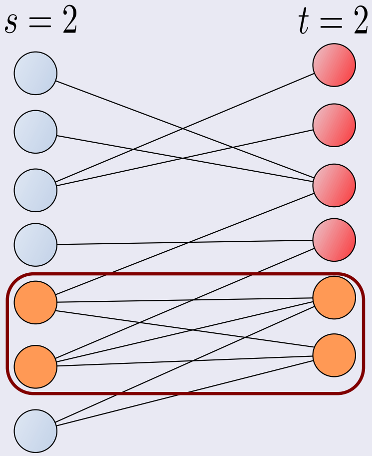
That is

If a set of t nodes on the right side is frequent, then they all occur together in at least s baskets



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The Basics

The set of all items in a market basket data is defined as

$$\mathcal{I} = \{i_1, i_2, ..., i_d\} \quad (1)$$

The set of all transactions (Baskets)

$$\mathcal{T} = \{t_1, t_2, ..., t_N\} \quad (2)$$

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Question

Now, we ask a really simplest question

Can you find sets of items that appear together "**frequently**" in the baskets?

Example of Frequent Itemsets

Items

Given a set $X = \{milk, coke, pepsi, beer, juice\}$

And the following baskets, we are looking the itemsets with support $s = 3$

$$B_1 = \{m, c, b\} \quad B_2 = \{m, p, j\} \quad B_3 = \{m, b\} \quad B_4 = \{c, j\}$$
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Can we do better?

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The idea is based on the following observations

1. If an itemset I does not satisfy the minimum support threshold, i.e. $support(I) < s \Rightarrow I$ is not frequent.
2. If an item A is added to the itemset I i.e. $\{A\} \cup I$, then the resulting itemset cannot occur more frequently than I.
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Proof

First, we prove that if itemset I is frequent then the subset are frequent

Given a transaction t_i, such that $I \subseteq t_i$, then for any subset $A \subseteq I \longrightarrow A \subseteq t_i$. Now as a result that $\sigma(I) \geq s$.

We can use the Monotonicity Property

Let I be a set of items, and $J = 2^I$ be the power set of I. A measure f is monotone if

$$\forall X, Y \in J \text{ if } X \subseteq Y \longrightarrow f(X) \leq f(Y) \quad (4)$$

Clearly

The cardinality is a monotone measure.

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Or

$$s < \sigma(I) \leq \sigma(A) \quad (6)$$

The itemset A is frequent.

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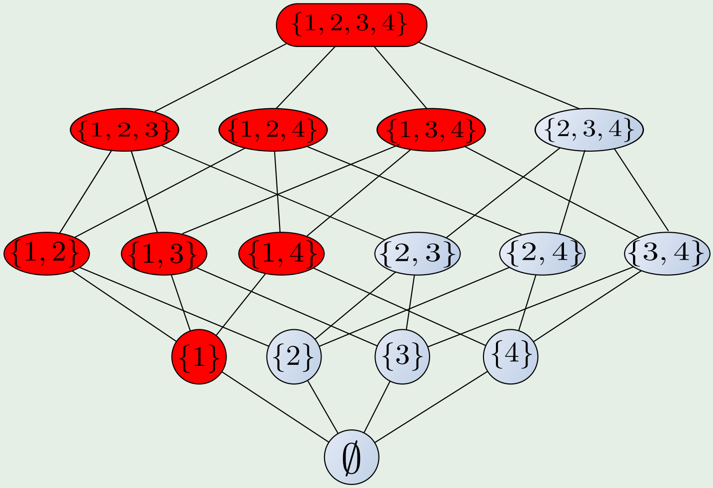
This principle allows to prune the power set

Example for $\{1\}$ not frequent



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The interest function is the difference between its confidence and the fraction of baskets that contain j

$$Interest(I \rightarrow \{j\}) = conf(I \rightarrow j) - Pr(\{j\})$$

Where

$$Pr(\{j\}) = \frac{|\{t_i | I \subseteq t_i, t_i \in T\}|}{\text{Numer of Baskets}} \quad (7)$$

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Interesting rules are those with high positive or negative interest values

For this, we have that

$$Pr[j] \gg conf\,(I \to j) \text{ or } conf\,(I \to j) \gg Pr[j] \quad (8)$$

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We measure the association rule $\{m, b\} \rightarrow c$

Thus, we have that

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  - Item c appears in $5/8$ of the baskets
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Find all association rules with support$\geq s$ and confidence$\geq c$

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We have a bunch of baskets

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Frequent itemsets

$\{b, m\} \{b, c\}$ $\{c, m\}$ $\{c, j\}$ $\{m, c, b\}$

Generate rules by eliminating anything below $c = 0.75$

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Maximal Frequent itemsets

No immediate superset is frequent

Closed itemsets

No immediate superset has the same count (> 0).

- It stores not only frequent information, but exact counts

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$\{A, B\}$	4	Yes	Yes
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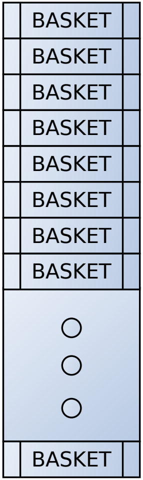
Thus

The File for the baskets

- It is stored on disk
- It is stored basket-by-basket

- You use k nested loops to generate all sets of size k



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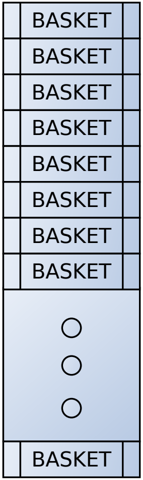

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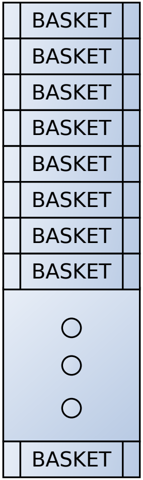

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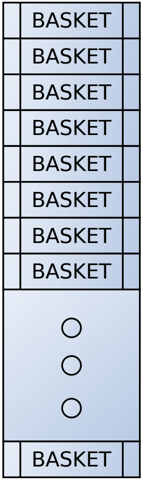

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Let us first concentrate on pairs, then extend to larger sets.

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The approach

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Fails if $(\text{Number of Items})^2$ exceeds main memory

Remember that the Number of Items can be $100\ Kb$ (Wal-Mart) or $10\ Gb$ (Web pages).

For example

Suppose we have 10^7 items and counts are 4-byte integers

Number of pairs of items

$$\frac{10^7\,(10^5\text{-}1)}{2} \approx 5 \times 10^{11} \tag{11}$$

Therefore, we need the following amount

$$4 \text{ bytes} \times 5 \times 10^{11} = 2 \times 10^{12} \text{ bytes} = 2 \text{ terabytes}$$

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- You can count all the pairs by simply using the counter at the cell $A[i, j] = A[i, j] + 1$.
- The storage used at this approach is 4 bytes per pair

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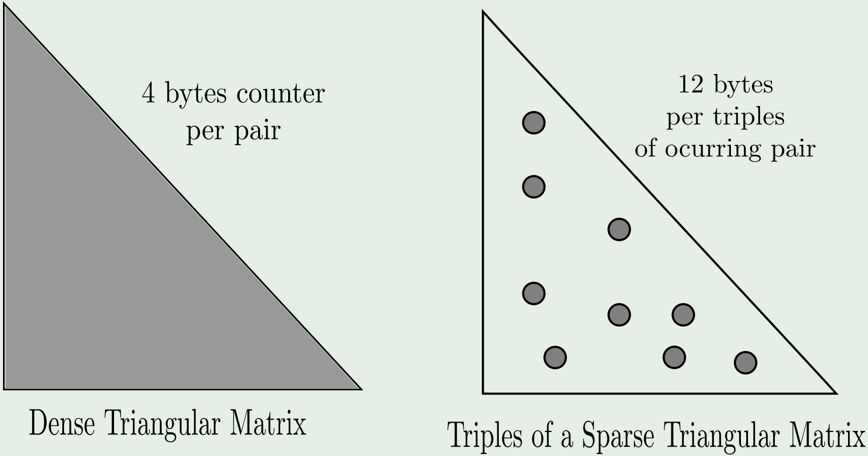
Comparing the 2 Approaches

Dense vs Sparse



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Storing Items in a Flat Array

- Keep pair counts in **lexicographic order:**
  - $\{1,2\}, \{1,3\}, ..., \{1,n\}, \{2,3\}, \{2,4\}, ..., \{2,n\}, \{3,4\}, ...$
- Pair $\{i, j\}$ is at position $(i{-}1)(n{-}i/2) + j{-}i$

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It is clear that

If we can store information in a hash table, we can really save memory.

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False Positive Counts can increase because of the nature of the hash table.

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Take this in consideration

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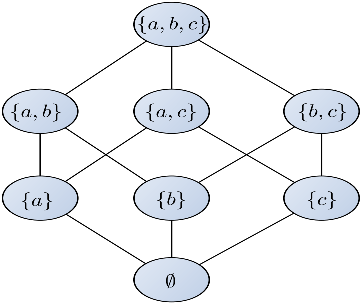
The main algorithm idea

- A two-pass approach called a-priori limits the need for main memory



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- If a set of items I appears at least s times, so does every **subset** J of I.

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- If item i does not appear in s baskets, then no pair including i can appear in s baskets

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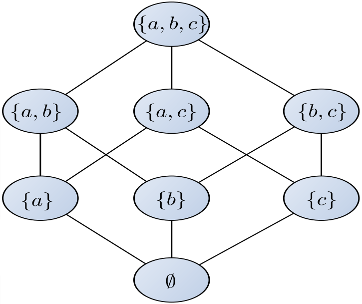

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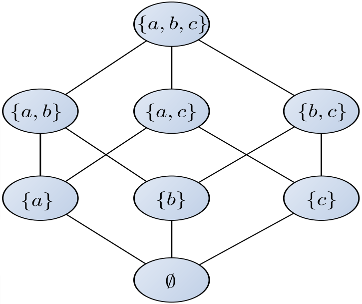

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Main-Memory Usage of the A-Priori Algorithm

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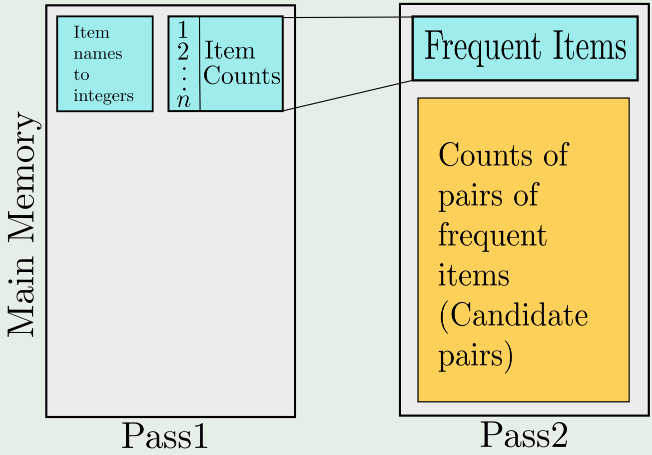

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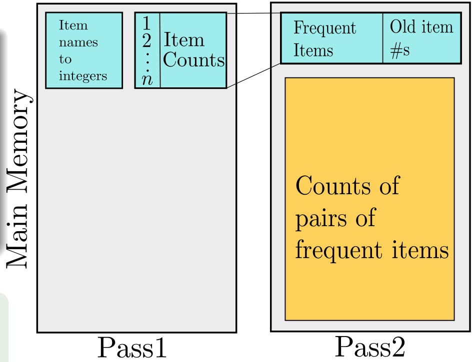

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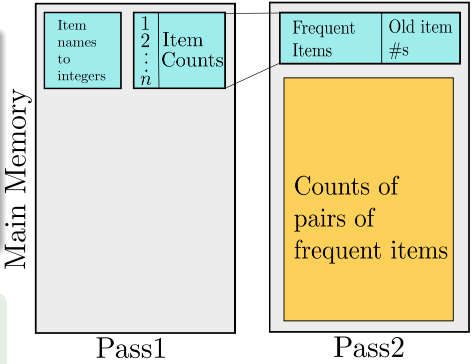

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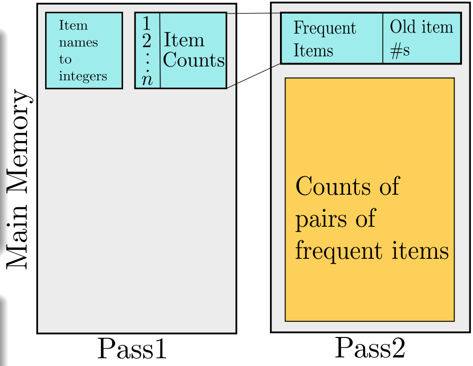

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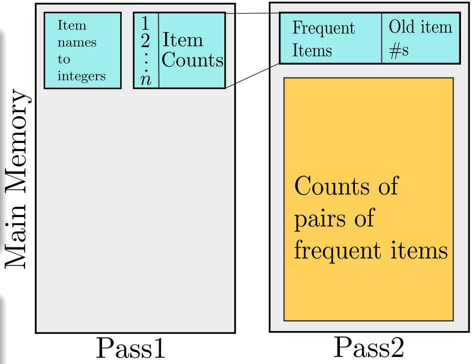

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Frequent Triples, Etc.

We have then the following procedure for k-tuples

- For each k, we construct two sets of k-tuples (sets of size k):
  - C_k = candidate k-**tuples** = those that might be frequent sets (support $> s$) based on information from the pass for k–1
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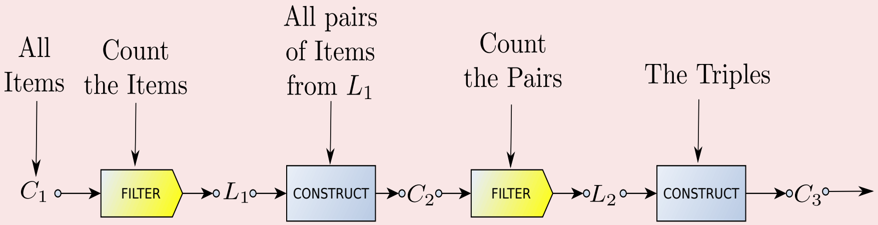

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Hypothetical steps of the A-Priori algorithm

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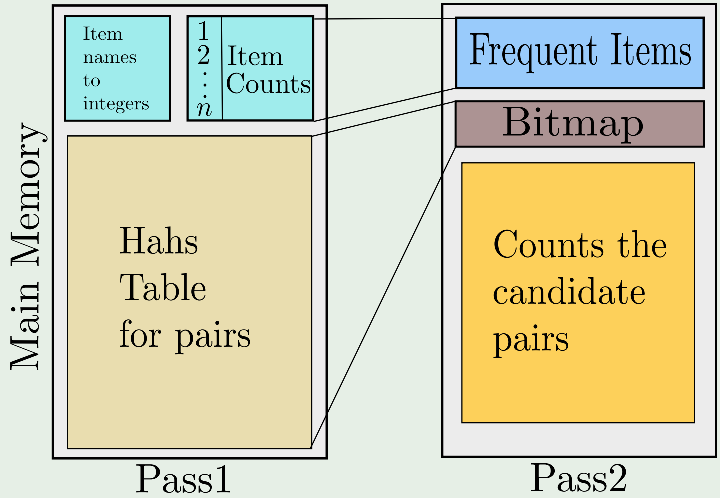

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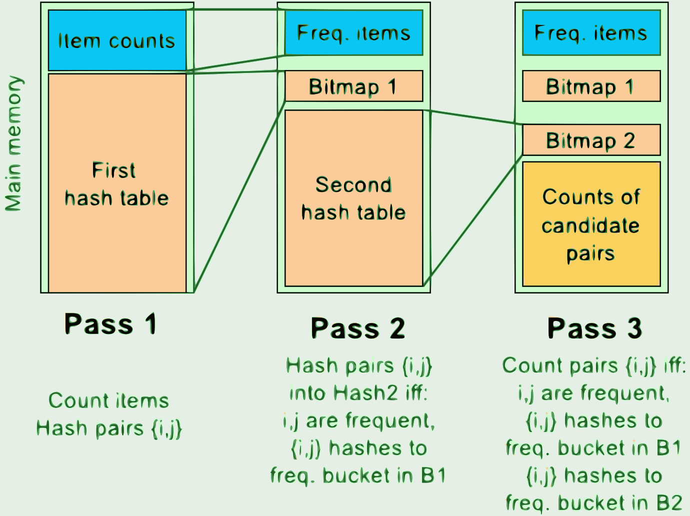

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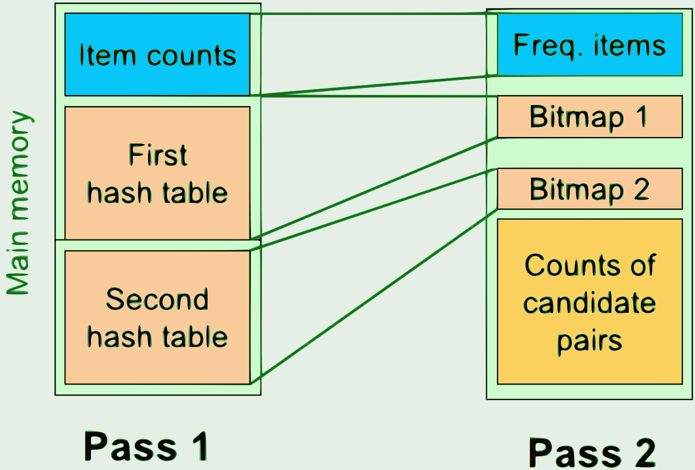

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Multistage or Multihash

- Either multistage or multihash can use more than two hash functions

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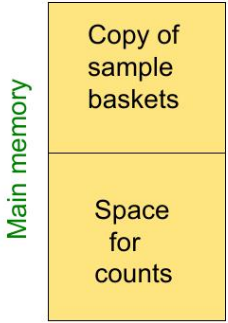

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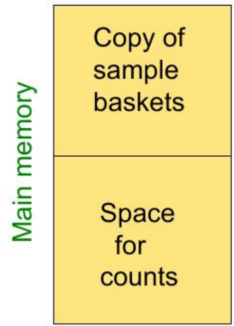

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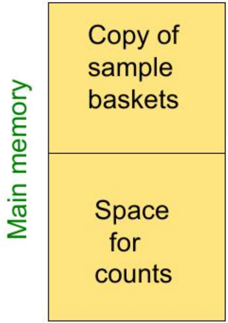

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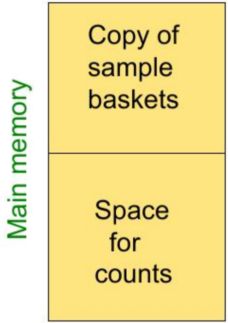

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