Analysis of Algorithms

Computational Geometry

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Outline

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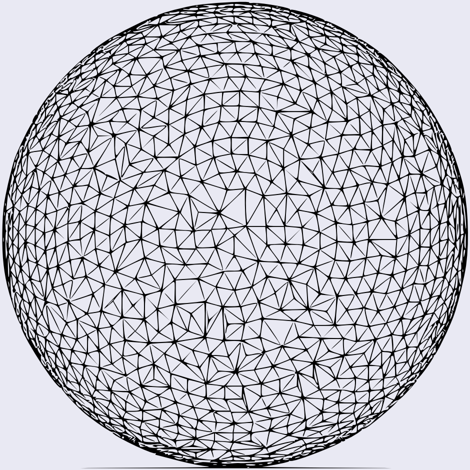
Computational Geometry

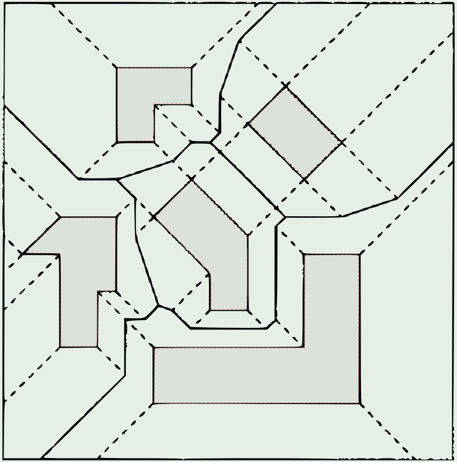
Motivation

- We want to solve geometric problems!!!



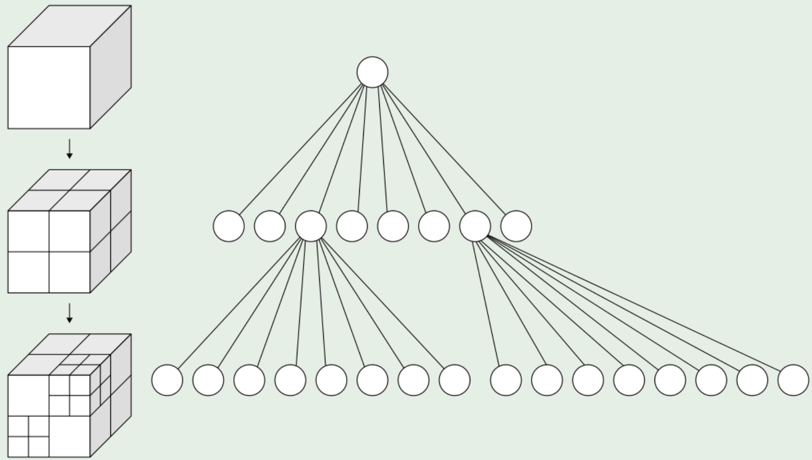
Field of Application

VLSI design - Generation for Fast Voronoi Diagrams for Massive Layouts Under Strict Distances to avoid Tunneling Effects!!



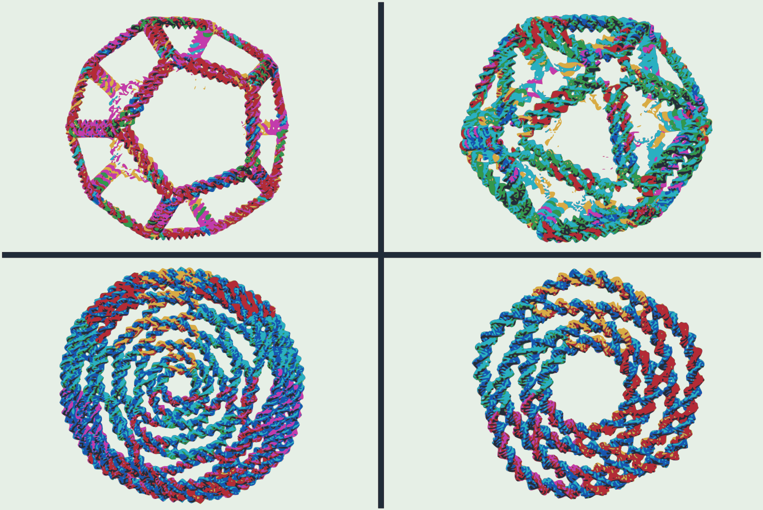
Field of Application

Databases - Octrees for fast localization of information in database tables



Field of Application

Synthetic Biology - Geometric Algorithms to Obtain new DNA configurations for Molecular Machines



Field of Application

Computer Graphics for more engaging Virtual Environments - For example: Bump Mapping!!!

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The Plane Representation

Although 3D algorithms exist...

- We will deal only with algorithms working in the plane.

  - $p_i = (x_i, y_i)$ and $x_i, y_i \in \mathbb{R}$

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Object Representation

- Each object is a set of points $\{p_1, p_2, ..., p_n\}$ where
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- For example an n-vertex polygon P is the following order sequence:
  - $\langle p_0, p_2, ..., p_n \rangle$

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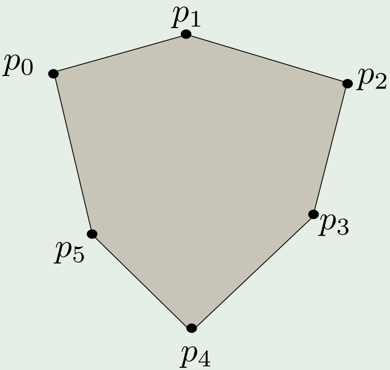
Example

Polygon



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Line-segment Properties

A convex combination

- Given two distinct points $p_1 = (x_1, y_1)^T$ and $p_2 = (x_2, y_2)^T$, a convex combination of $\{p_1, p_2\}$ is any point p_3 such that:
  - $p_3 = \alpha p_1 + (1 - \alpha) p_2$ with $0 \leq \alpha \leq 1$.

- If $p_1 = (0, 0)$ then $\overrightarrow{p_1 p_2}$ is the vector p_2.

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Line Segment as Convex Combination

- Given two points p_1 and p_2 (Known as End Points), the line segment $\overline{p_1 p_2}$ is the set of convex combinations of p_1 and p_2.

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Directed Segment

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Cross Product

Question!!!

- Given two directed segments $\overrightarrow{p_0 p_1}$ and $\overrightarrow{p_0 p_2}$,
  - Is $\overrightarrow{p_0 p_1}$ clockwise from $\overrightarrow{p_0 p_2}$ with respect to their common endpoint p_0?

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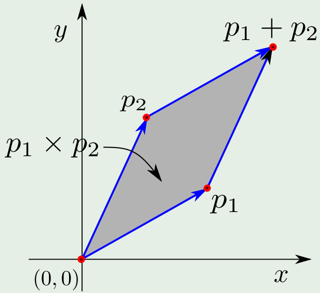

Cross Product

A shorter representation

$$p_1 \times p_2 = \det \begin{pmatrix} p_1 & p_2 \end{pmatrix} = \det \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} = x_1 y_2 - x_2 y_1 = -p_2 \times p_1$$

- If $p_1 \times p_2$ is negative, then p_1 is counterclockwise from p_2.

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Thus

- if $p_1 \times p_2$ is positive, then p_1 is clockwise from p_2.
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Regions

Clockwise and Counterclockwise Regions

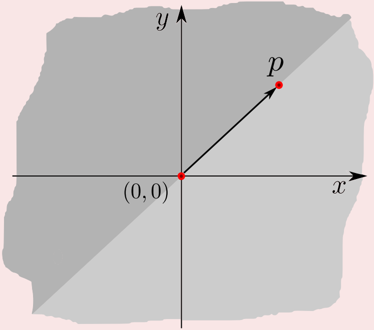

Figure: Darker counterclockwise; lighter clockwise with respect to p

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Turn Left or Right

Question

Given two line segments $\overrightarrow{p_0 p_1}$ and $\overrightarrow{p_1 p_2}$,

- if we traverse $\overrightarrow{p_0 p_1}$ and then $\overrightarrow{p_1 p_2}$, do we make a left turn at point p_1?

Turn Left or Right

Simply use the following idea

- Compute cross product $(p_2 - p_0) \times (p_1 - p_0)$!!!
- **This translates p_0 to the origin!!!**
- What about $(p_2 - p_0) \times (p_1 - p_0) = 0$?

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Left Turn = counterclockwise; Right Turn = clockwise

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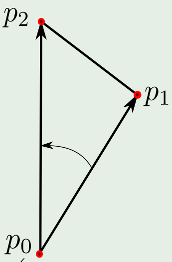
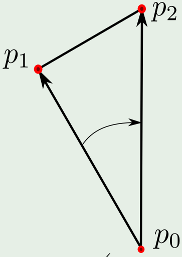
counterclockwise



$$(p_2 - p_0) \times (p_1 - p_0) = \begin{pmatrix} x_2 - x_0 & x_1 - x_0 \\ y_2 - y_0 & y_1 - y_0 \end{pmatrix} < 0$$

clockwise



$$(p_2 - p_0) \times (p_1 - p_0) = \begin{pmatrix} x_2 - x_0 & x_1 - x_0 \\ y_2 - y_0 & y_1 - y_0 \end{pmatrix} > 0$$

Code for this

We have the following code

Direction(p_i, p_j, p_k)

1. return $(p_k - p_i) \times (p_j - p_i)$

Outline

Intersection

Question

Do line segments $\overrightarrow{p_1 p_2}$ and $\overrightarrow{p_3 p_4}$ intersect?

- An endpoint of one segment lies on the other segment

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Very Simple!!! We have two possibilities

1. Each segment straddles the line containing the other.
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Case I This summarize the previous two possibilities

The segments straddle each other's lines.

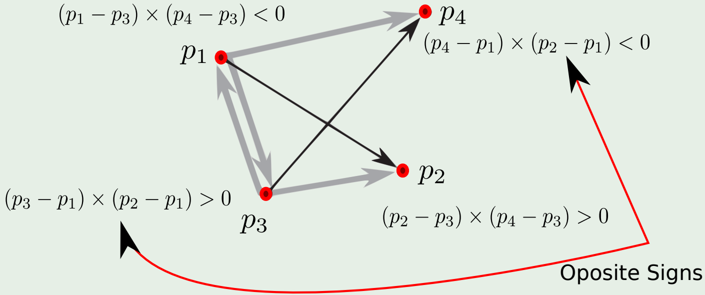

Figure: Using Cross Products to find intersections

Case II No intersection

The segment straddles the line, but the other does not straddle the other line

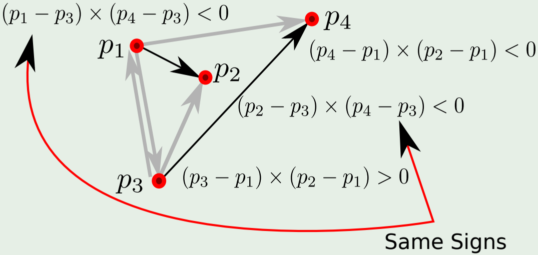

Figure: Using Cross Products to find that there is no intersection

Code

Code

Segment-Intersection(p_1, p_2, p_3, p_4)

1. $d_1 = Direction\,(p_3, p_4, p_1)$
2. $d_2 = Direction\,(p_3, p_4, p_2)$
3. $d_3 = Direction\,(p_1, p_2, p_3)$
4. $d_4 = Direction\,(p_1, p_2, p_4)$
5. **if** $((d_1 > 0$ **and** $d_2 < 0)\ or\ (d_1 < 0$ **and** $d_2 > 0)\ and$
6. $\qquad (d_3 > 0$ **and** $d_4 < 0)\ or\ (d_3 < 0$ **and** $d_4 > 0))$
7. $\qquad\qquad$ **return** $TRUE$

Figure: The Incomplete Code, You still need to test for endpoints over the segment

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Sweeping

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Use an imaginary vertical line to pass through the n segments with events $x \in \{r, t, u\}$:

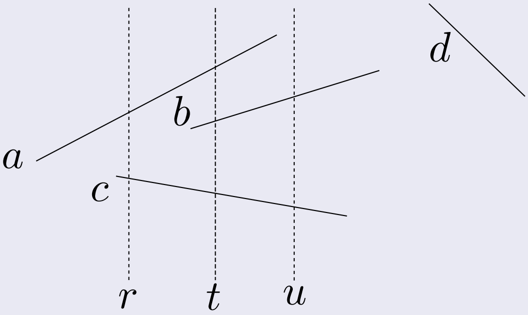

Figure: Vertical Line to Record Events

Thus

This can be used to record events given two segments s_1 and s_2

- Event I: s_1 above s_2 at x, written $s_1 \succsim_x s_2$.
  - This is a total preorder relation for segment intersecting the line at x.
  - The relation is transitive and reflexive.
- Event II: s_1 intersect s_2, then neither $s_1 \succsim_x s_2$ or $s_2 \succsim_x s_1$, or both (**if** s_1 **and** s_2 **intersect at** x)

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Example

Example: $a \succcurlyeq_r c \;\; a \succcurlyeq_t c$

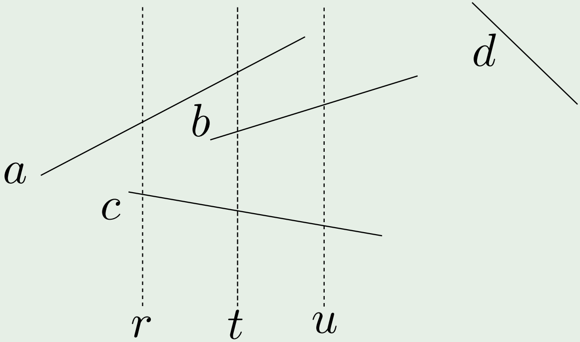
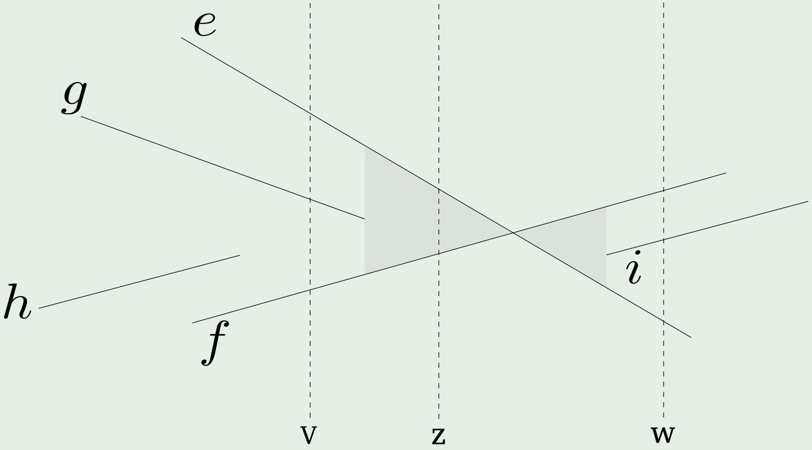


Figure: Vertical Line to Record Events

Change in direction

When e and f intersect, $e \succsim_v f$ and $f \succsim_w e$. In the Shaded Region, any sweep line will have e and f as consecutive



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Moving the sweep line

Something Notable

Sweeping algorithms typically manage two sets of data.

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Sweep-line status

The sweep-line status gives the relationships among the objects that the sweep line intersects.

Event-point schedule

The event-point schedule is a sequence of points, called event points, which we order from left to right according to their x-coordinates.

- As the sweep progress from left to right, it stops and processes each event point, then resumes.
- It is possible to use a min-priority queue to keep those event points sorted by x-coordinate.

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Sweeping Process

First

- We sort the segment endpoints by increasing x-coordinate and proceed from left to right.

However, sometimes they have the same x-coordinate (Covertical)

If two or more endpoints are covertical, we break the tie by putting all the covertical left endpoints before the covertical right endpoints.

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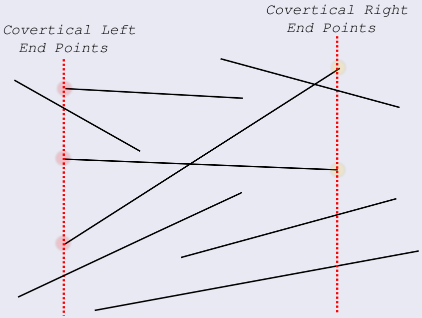
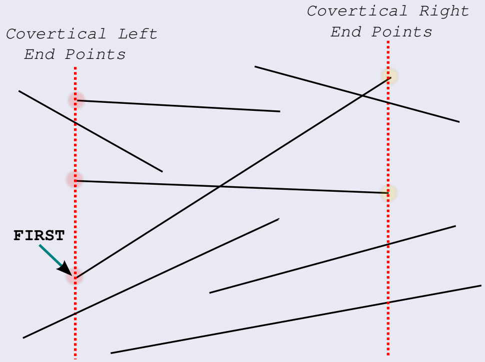


Then

Second

Within a set of covertical left endpoints, we put those with lower y-coordinates first, and we do the same within a set of covertical right endpoints.



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Process

1. When we encounter a segment's left endpoint, we insert the segment into the sweep-line status.
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Operations

Operations to keep preorder on the events for algorithm

- INSERT(T, s): insert segment s into T.
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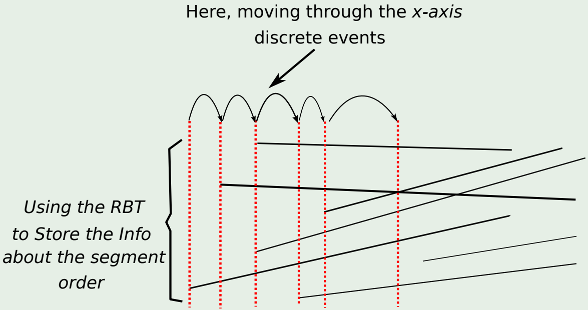
What the algorithm does?

Moving the sweeping line discretely - Event-point schedule



Event-Point Schedule Implementation

For this

We can use a Priority Queue using lexicographic order

The interesting part is the Sweeping-Line Satus

Because the way we build the balanced tree

Event-Point Schedule Implementation

For this

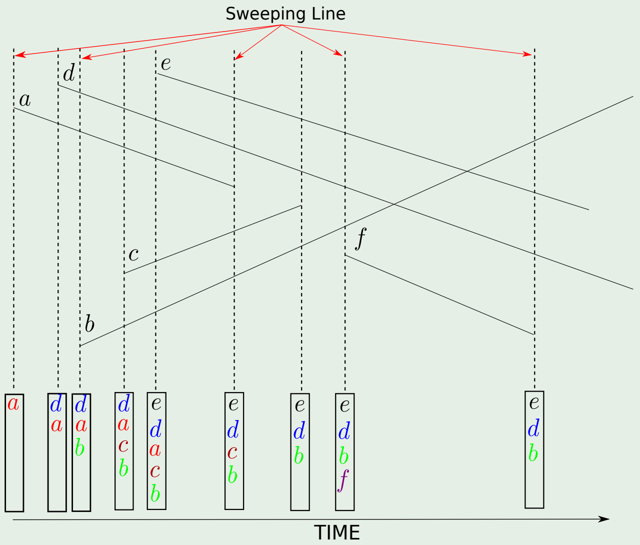
We can use a Priority Queue using lexicographic order

The interesting part is the Sweeping-Line Satus

Because the way we build the balanced tree

Sweep-Line Status

The Above and Below relation



Sweeping Line Status Implementation

Use the following relation of order to build the binary tree

Given a segment x, then you insert y

- Case I if y is counterclockwise, it is below x (Go to the left).
- Case II if y is clockwise, it is above x (Go to the Right)

In addtion

If you are at a leaf do the insertion, but also insert the leaf at the left or right given the insertion.

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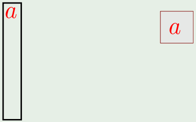
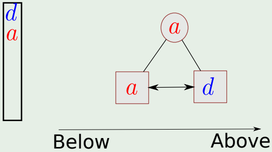
Example

We insert th first element in the circular leaves list



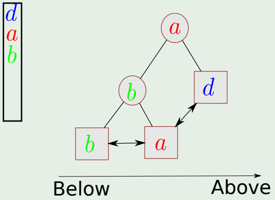
Example

We insert a inner node after binary search



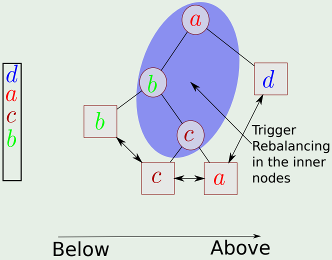
Example

Similar



Example

Etc....


d
a
c
b
a
b
d
b
c
c
a
Trigger
Rebalancing
in the inner
nodes
Below
Above

Pseudo-code with complexity $O\left(n \log_2 n\right)$

Any-Segment-Intersect(S)

1. $T = \emptyset$
2. Sort the endpoints of the segments in S from left to right
   Breaking ties by putting left endpoints before right endpoints
   and breaking further ties by putting points with lower
   $y-$coordinates first
3. **for each** point p in the sorted list
4. **if** p is the left endpoint of a segment s
5. INSERT(T, s)
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Outline

Cinvestav

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The ANY-SEGMENTS-INTERSECT returns TRUE

If it finds an intersection between two of the input segments.

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Then, for a and b

- Since no intersections occur to the left of p, the order given by T (Sweeping Line Data Structure) is correct at all points to the left of p.
- Assuming that no three segments intersect at the same point, a and b become consecutive in the total preorder of some sweep line z.

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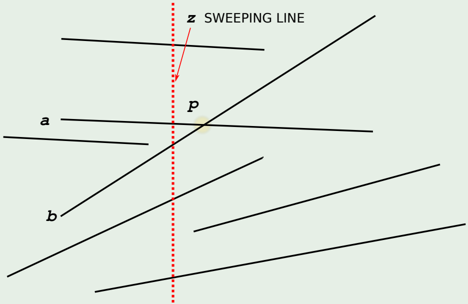
Now, we have two possbibilities

Case I



Case I

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z is to the left of p or goes through p.

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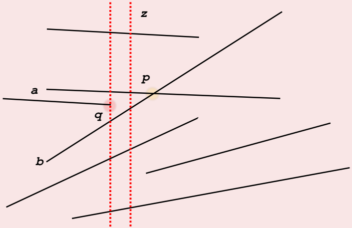


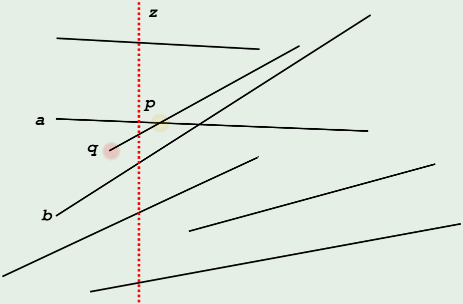
Finally

Then a and b

They become consecutive in the total pre-order of a sweep line.

Case II

We have that q is a left endpoint where a and b stop being consecutive



Correctness about the order given by T

Then, given the two following cases

1. if p is in the sweep line $\Rightarrow p == q$.
2. If q is at the left of p, and it is the nearest left one.

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Do we mantain the correct preorder?

We have that given that p is first

Then, it is processed first becacuse the lexicographic order.

Therefore, two cases can happen

1. The point is processed - then the algorithm returns true
2. If the event is not processed - then the algorithm must have returned true

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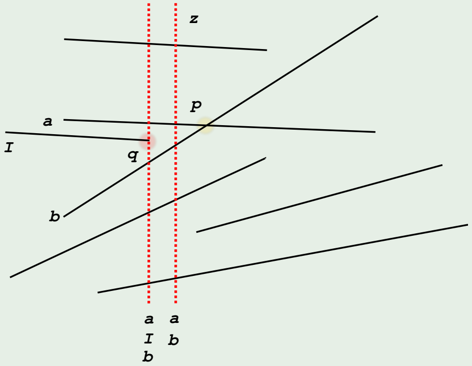
Handling Case I

Segments a and b are already in T , and a segment between them in the total pre-order is deleted, making a and b to become consecutive



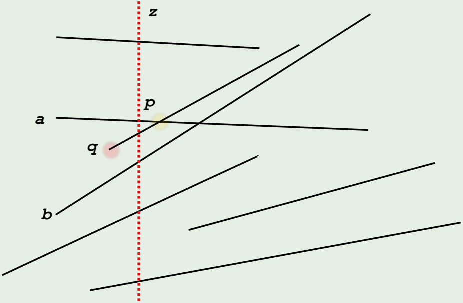
When is this detected?

In the following lines of the code

Lines 8–11 detect this case.

Handling Case II

Either a or b is inserted into T, and the other segment is above or below it in the total pre-order.



When is this detected?

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Lines 4–7 detect this case.

Finally

If event point q is not processed

It must have found an earlier intersection!!!

Therefore

If there is an intersection Any-Segment-Intersect returns true all the time

Cinvestav

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Running Time

Something Notable

1. Line 1 takes $O(1)$ time.
2. Line 2 takes $O(n \log_2 n)$ time, using merge or heap sort
3. The for loop iterates at most $2n$ times
   1. Each iteration takes $O(\log_2 n)$ in a well balanced tree.
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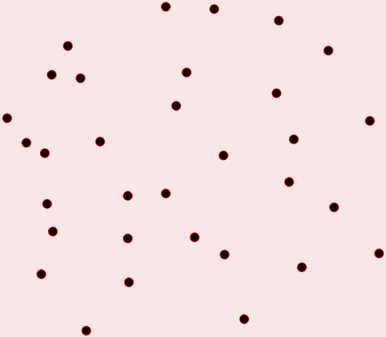
Cinvestav

Convex Hull

Convex Hull

- Given a set of points, Q, find the smallest convex polygon P such that $Q \subset P$. This is denoted by $\text{CH}(Q)$.

Bunch of Points in 2D



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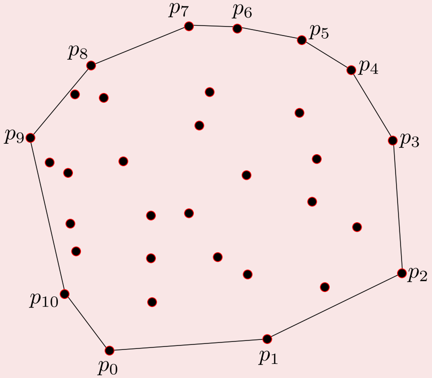

Convex Hull

The two main Algorithms (Using the "Rotational Sweep") that we are going to explore

- Graham's Scan.

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Nevertheless there are other methods

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Graham's Scan Basics

- **It keeps a Stack of candidate points.**
- It Pops elements that are not part of the CH(Q).
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Graham's Scan Code

Algorithm

GRAHAM-SCAN(Q)

1. Let p_0 be the point Q with the minimum y-coordinate or the leftmost such point in case of a **tie**
2. let $\langle p_1, p_2, ..., p_n \rangle$ be the remaining points in Q, sorted by polar angle in counter clockwise order around p_0 (If more than one point has the same angle, remove all but one that is farthest from p_0)
3. Let S be an empty stack
4. PUSH(p_0, S)
5. PUSH(p_1, S)
6. PUSH(p_2, S)
7. **for** $i = 3$ **to** n
8.   **while** ccw(next-top(S), p_i, top (S)) ≤ 0
9.   POP(S)
10.   PUSH(S)
11. **return** S

▷ The clockwise and counter clockwise algorithm◁

ccw(p_1, p_2, p_3)

1. return $(p_3 - p_1 \times p_2 - p_1)$

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2. let $\langle p_1, p_2, ..., p_n \rangle$ be the remaining points in Q, sorted by polar angle in counter clockwise order around p_0 (If more than one point has the same angle, remove all but one that is farthest from p_0)
3. Let S be an empty stack
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7. **for** $i = 3$ **to** n
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▷ The clockwise and counter clockwise algorithm◁

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Graham's Scan Code

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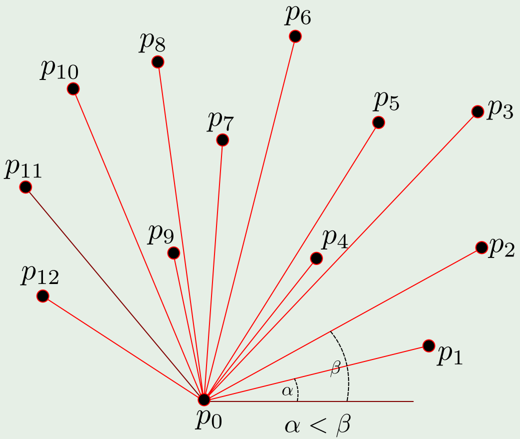
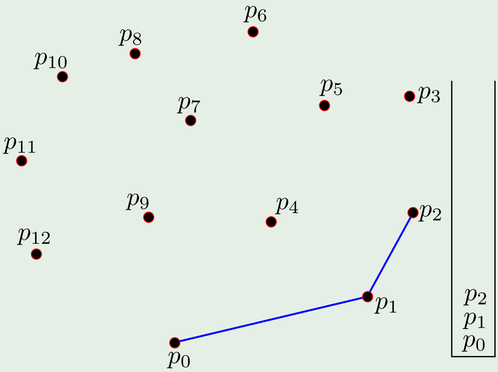
Example

Sort points using the smallest to largest polar coordinate



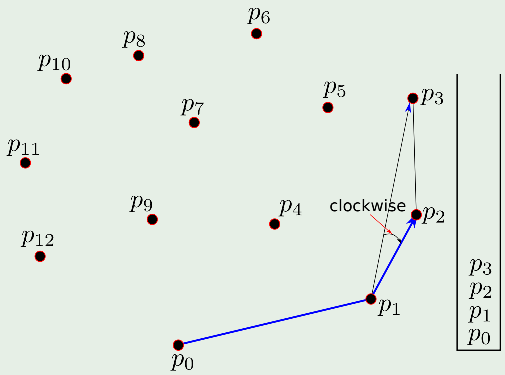
Example

Push the first points into the stack



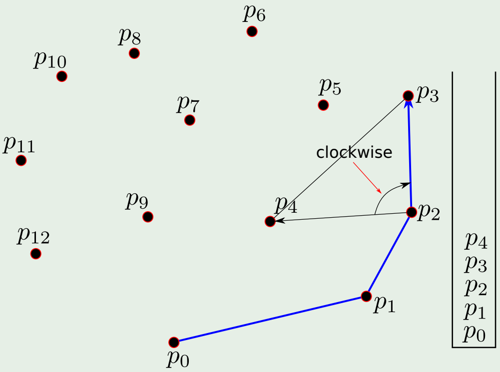
Example

ccw$(p_1, p_2, p_3) > 0$, do not get into the loop and push p_3 into S



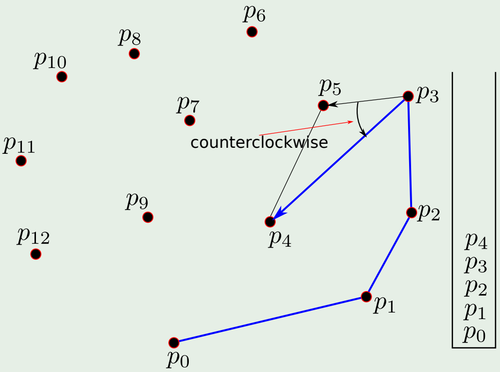
Example

ccw$(p_2, p_3, p_4) > 0$, do not get into the loop and push p_4 into S



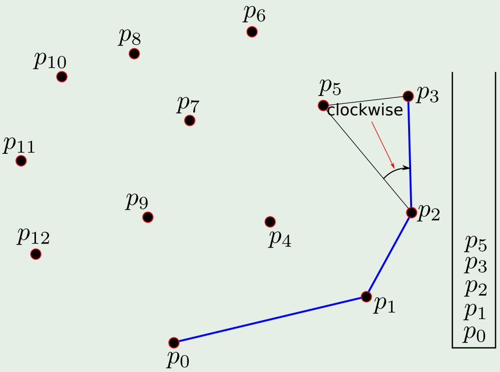
Example

Counterclockwise - Pop p_4



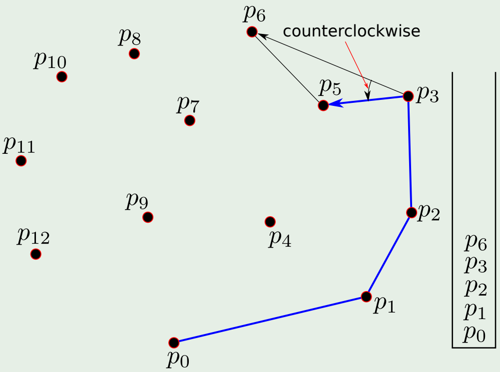
Example

Clockwise - Push p_5



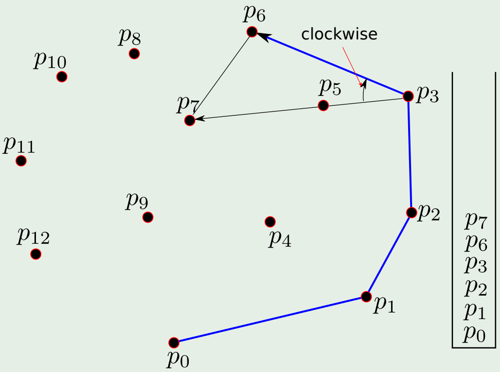
Example

Counterclockwise - Pop p_5 and push p_6



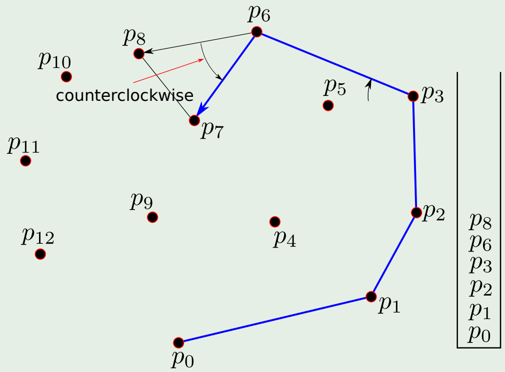
Example

Clockwise push p_7



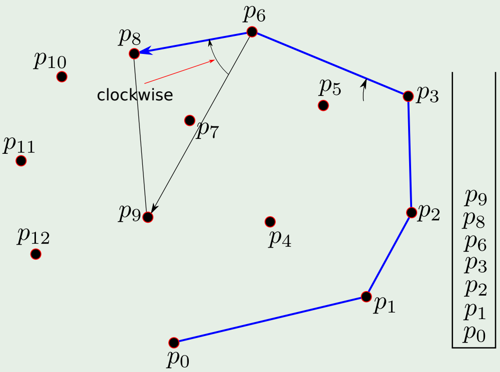
Example

Counterclockwise pop p_7 and push p_8



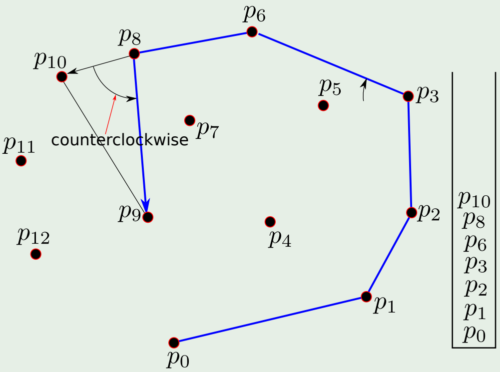
Example

Clockwise push p_9



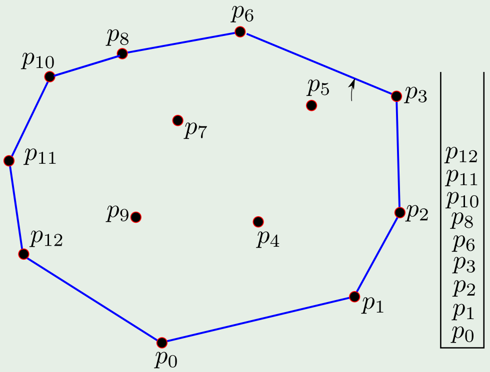
Example

Clockwise pop p_9 and push p_{10}



Example

Keep going until you finish



We have the following theorem for correctness of the algorithm

Theorem 33.1 (Correctness of Graham's scan)

If GRAHAM-SCAN executes on a set Q of points, where $|Q| \geq 3$, then at termination, the stack S consists of, from bottom to top, exactly the vertices of CH(Q) in counterclockwise order.

The proof is based in loop invariance

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Using Aggregate Analysis to obtain the complexity $O\left(n \log_2 n\right)$

Complexity

1. Line 2 takes $O\left(n \log_2 n\right)$ using Merge sort or Heap sort by using polar angles and cross product.
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PUSH takes $O(1)$ time:

- Each iteration takes $O\left(1\right)$ time not taking in account the time spent in the **while** loop in lines 8-9.

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We have that

For $i = 0, 1, ..., n$, we push each point p_i into the stack S exactly once.

Remember Multipop?

We can pop at most the number of items that we push on it.

Thus

At least three points p_0, p_1 and p_m are never popped out of the stack!!!

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We have $m-2$ POP operations are performed in total!!! If we had pushed m elements into S.

Thus, each iteration of the while loop

It performs one POP, and there are at most $m-2$ iterations of the **while** loop altogether.

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Outline

Jarvis' March

Jarvis' March Basics

- It computes CH by using
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  - At each point calculate the minimum polar angle.
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Jarvis's march builds a sequence

$H = \langle p_0, p_1, p_2, ..., p_{h-1} \rangle$ of the vertices of CH(Q)

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When we reach the highest vertex, p_k (Breaking ties by choosing the farthest such vertex), we have constructed **the right chain** of CH(Q).

To construct **the left chain**

We start at p_k, then we choose p_{k+1} as the point with the smallest polar angle with respect to p_k negative , but from ***the negative x-axis***.

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Example

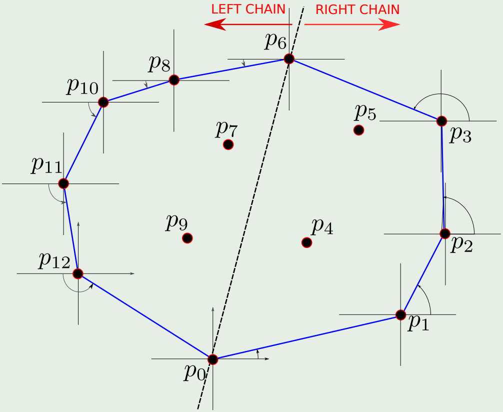

Figure: Wrapping the Gift. Here the Right Chain finishes at p_6, then the Left Chain is started

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Something Notable

Complexity $O\left(hn\right)$

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