Analysis of Algorithms

String matching

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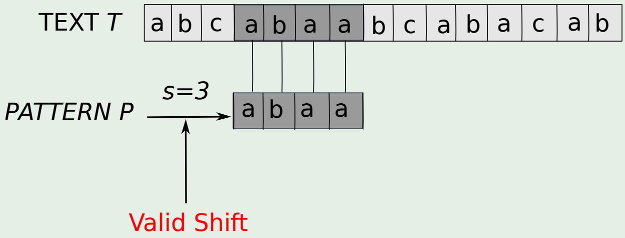
Outline

1. String Matching
   - Introduction
   - Some Notation
2. Naive Algorithm
   - Using Brute Force
3. The Rabin-Karp Algorithm
   - Efficiency After All
   - Horner's Rule
   - Generaiting Possible Matches
   - The Final Algorithm
   - Other Methods
4. Exercises

What is string matching?

Given two sequences of characters drawn from a finite alphabet Σ, $T[1..n]$ and $P[1..m]$



Where...

A Valid Shift

P occurs with a ***valid shift*** s if for some $0 \leq s \leq n - m \Longrightarrow T[s+1..s+m] == P[1..m]$.

Otherwise

it is an invalid shift.

Thus

The string-matching problem is the problem of of finding all valid shifts given a patten P on a text T.

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Possible Algorithms

We have the following ones

Algorithm	Preprocessing Time	Worst Case Matching Time
Naive	0	$O\left(\left(n-m+1\right)m\right)$
Rabin-Karp	$\Theta\left(m\right)$	$O\left(\left(n-m+1\right)m\right)$
Finite Automaton	$O\left(m\left\|\Sigma\right\|\right)$	$\Theta\left(n\right)$
Knuth-Morris-Pratt	$\Theta\left(m\right)$	$\Theta\left(n\right)$

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Notation and Terminology

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- The ε is both suffix and prefix of every string.

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The naive string algorithm

Algorithm

NAIVE-STRING-MATCHER(T, P)

1. $n = T.length$
2. $m = P.length$
3. for $s = 0$ to $n - m$
4.   if $P[1..m] == T[s+1..s+m]$
5.   print "Pattern occurs with shift" s

Complexity

$O((n-m+1)m)$ or $\Theta(n^2)$ if $m = \lfloor \frac{n}{2} \rfloor$

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A more elaborated algorithm

Rabin-Karp algorithm

Lets assume that $\Sigma = \{0, 1, ..., 9\}$ then we have the following:

- Thus, each string of k consecutive characters is a k-length decimal number:

$$c_1 c_2 \cdots c_{k-1} c_k = 10^{k-1} c_1 + 10^{k-2} c_2 + \ldots + 10 c_{k-1} + c_k$$

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Thus

- p correspond the decimal number for pattern $P[1..m]$.
- t_s denote decimal value of m-length substring $T[s+1..s+m]$ for $s = 0, 1, ..., n-m$.

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- t_s denote decimal value of m-length substring $T[s+1..s+m]$ for $s = 0, 1, ..., n-m$.

Then

Properties

Clearly $t_s == p$ if and only if $T[s+1..s+m] == P[1..m]$, thus s is a valid shift.

Now, think about this

What if we put everything in a single word of the machine

- If we can compute p in $\Theta(m)$.
- If we can compute all the t_s in $\Theta(n-m+1)$.

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We will get

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Remember Horner's Rule

Consider

- We can compute, the decimal representation by the Horner's rule:

$$p = P[m] + 10(P[m-1] + 10(P[m-2] + ... + 10(P[2] + 10P[1])...)) \tag{1}$$

Thus, we can compute t_0 using this rule in

$$\Theta(m) \tag{2}$$

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Example

If you have the following set of digits

2	4	0	1

Using the Horner's Rule for $m = 4$

$$
\begin{aligned}
2401 &= 1 + 10 \times (0 + 10 \times (4 + 10 \times 2)) \\
&= 2 \times 10^3 + 4 \times 10^2 + 0 \times 10 + 1
\end{aligned}
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Then

To compute the remaining values, we can use the previous value

$$t_{s+1} = 10\left(t_s - 10^{m-1}T\left[s+1\right]\right) + T\left[s+m+1\right]. \quad (3)$$

Notice the following

- Subtracting from it $10^{m-1}T\left[s+1\right]$ removes the high-order digit from t_s.

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Example

Imagine that you have...

index	1	2	3	4	5	6
digit	1	0	2	4	1	0

1. We have $t_0 = 1024$ then we want to calculate 0241
2. Then we subtract $\left(10^3 \times T[1]\right) == 1000$ of 1024
3. We get 0024
4. Multiply by 10, and we get 0240
5. We add $T[5] == 1$
6. Finally, we get 0241

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Remarks

First

We can extend this beyond the decimals to any d digit system!!!

What happens when the numbers are quite large?

We can use the module

Meaning

- Compute p and t_s values modulo a suitable modulus q.

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Remember Hash Functions

Yes, we are mapping the large numbers into the set

$$\{0, 1, 2, ..., q - 1\}$$

Then, to reduce the possible representation

Use the module of q

It is possible to compute $p \mod q$ in $\Theta(m)$ time and all the $t_s \mod q$ in $\Theta(n-m+1)$ time.

Something Notable

If we choose the modulus q as a prime such that $10q$ just fits within one computer word, then we can perform all the necessary computations with single-precision arithmetic.

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Why $10q$?

After all

$10q$ is the number that will subtracting for or multiplying by!!!

We use "truncated division" to implement the modulo operation

For example given two numbers a and n, we can do the following

$$q = trunc\left(\frac{a}{n}\right)$$

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The following

$$t_{s+1} = \left(d\left(t_s - T[s+1]h\right) + T[s+m+1]\right) \mod q \quad (4)$$

where $h \equiv d^{m-1} (\text{mod } q)$ is the value of the digit "1" in the high-order position of an m-digit text window.

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We have a small problem!!!

Question?

Can we differentiate between $p \mod q$ and $t_s \mod q$?

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- $25 \mod 11 == 3$

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cinvestav
Unidad Guadalajara

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The Final Algorithm

Rabin-Karp-Matcher(T, P, d, q)

$n = T.length$

$m = P.length$

$h = d^{m-1} \mod q$ // Storing the reminder of the highest power

$p = 0$

$t_0 = 0$

for $i = 1$ **to** m // Preprocessing

    $p = (dp + P[i]) \mod q$

    $t_0 = (dp + T[i]) \mod q$

for $s = 0$ **to** $n - m$

    if $p == t_s$

        if $P[1..m] == T[s+1..s+m]$ // Actually a Loop

            print "Pattern occurs with shift" s

    if $s < n - m$

        $t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) \mod q$

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t_0 = 0
for i = 1 to m // Preprocessing
    p = (dp + P[i]) mod q
    t_0 = (dp + T[i]) mod q
for s = 0 to n - m
    if p == t_s
        if P[1..m] == T[s+1..s+m] // Actually a Loop
            print "Pattern occurs with shift" s
    if s < n - m
        t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) mod q
```

The Final Algorithm

Rabin-Karp-Matcher(T, P, d, q)

$n = T.length$

$m = P.length$

$h = d^{m-1} \mod q$ // Storing the reminder of the highest power

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Preprocessing

1. $p \mod q$ is done in $\Theta(m)$.
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Then checking $P[1..m] == T[s+1..s+m]$

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The number of spurious hits is $O\left(n/q\right)$.

Because

We can estimate the chance that an arbitrary t_s will be equivalent to $p \mod q$ as $1/q$.

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Since there are $O\left(n\right)$ positions at which the test of line 10 fails (Thus, you have $O\left(n/q\right)$ non valid hits) and we spend $O\left(m\right)$ time per hit

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$$O(n) + O\left(m\left(v + \frac{n}{q}\right)\right)$$

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Outline

1. String Matching
   - Introduction
   - Some Notation
2. Naive Algorithm
   - Using Brute Force
3. The Rabin-Karp Algorithm
   - Efficiency After All
   - Horner's Rule
   - Generaiting Possible Matches
   - The Final Algorithm
   - Other Methods
4. Exercises

We have other methods

We have the following ones

Algorithm	Preprocessing Time	Worst Case Matching Time
Rabin-Karp	$\Theta(m)$	$O((n-m+1)m)$
Finite Automaton	$O(m\|\Sigma\|)$	$\Theta(n)$
Knuth-Morris-Pratt	$\Theta(m)$	$\Theta(n)$

Remarks about Knuth-Morris-Pratt

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It is quite an elegant algorithm that improves over the state machine.

How?

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- It encapsulate information how the pattern matches against shifts of itself

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Boyer–Moore string search algorithm

It was presented at the same time

It is used in the GREP function for pattern matching in UNIX

Actually is the basic algorithm to beat when doing research in this area!!!

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He gave a a proof of the algorithm with an upper bound of $3m$ comparisons in the worst case!!!

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Exercises

- 32.1-1
- 32.1-2
- 32.1-4
- 32.2-1
- 32.2-2
- 32.2-3
- 32.2-4