Analysis of Algorithms

Basic Graph Algorithms

Andres Mendez-Vazquez

October 28, 2015

Outline

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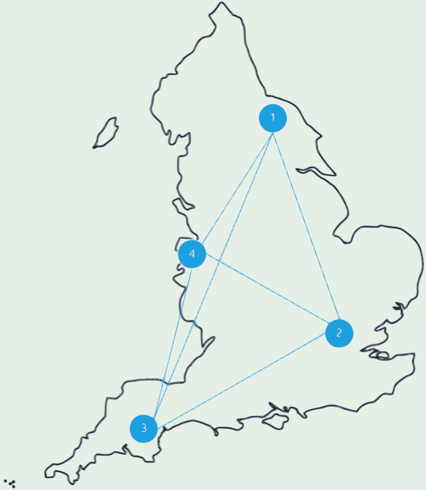
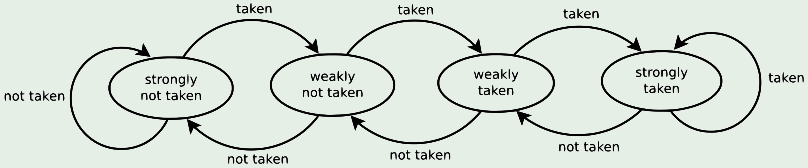
We are full of Graphs

Maps



We are full of Graphs

Branch CPU estimators



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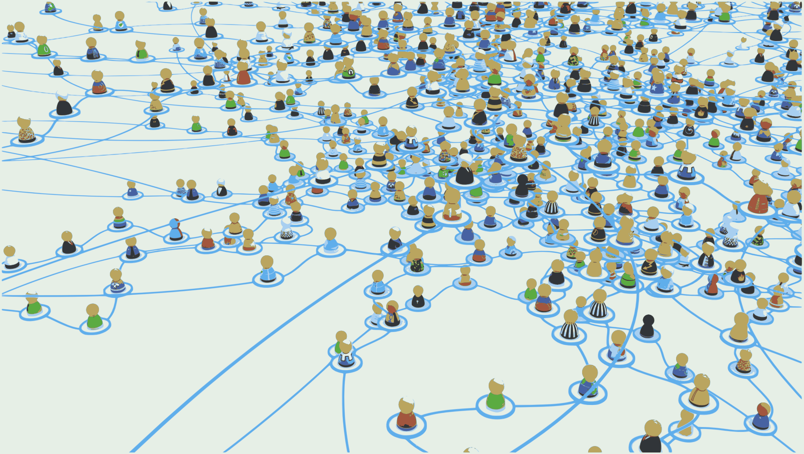
Social Networks



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History

Something Notable

Graph theory started with Euler who was asked to find a nice path across the seven Königsberg bridges

The Actual City

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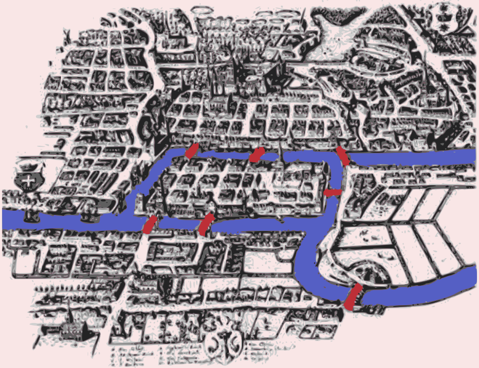

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What we want

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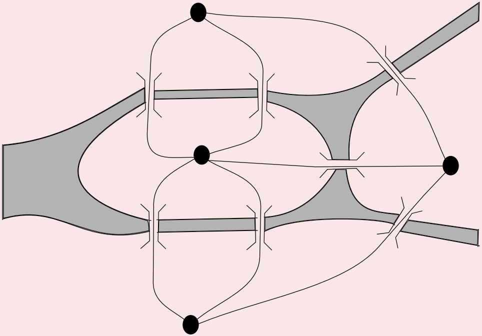

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A necessary condition for the walk of the desired form is that the graph be connected and have exactly zero or two nodes of odd degree.

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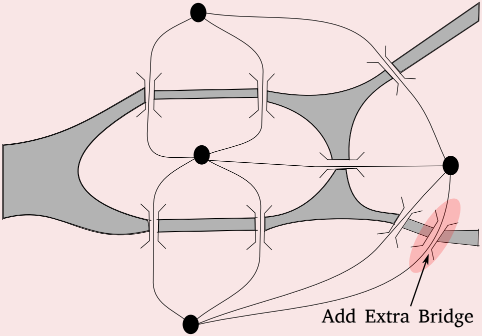

Studying Graphs

All the previous examples are telling us

Data Structures are required to design structures to hold the information coming from graphs!!!

Good Representations

They will allow to handle the data structures with ease!!!

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Definition

A Graph is composed of the following parts: **Nodes** and **Edges**

Nodes

They can represent multiple things:

- Distance between cities
- Friendships
- Matching Strings
- Etc

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Example

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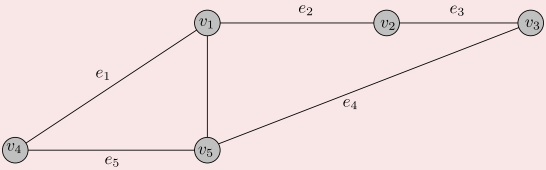

Properties

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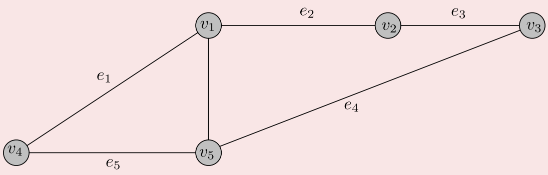

Some properties

Degree

The degree $d(v)$ of a vertex V is its number of incident edges

A self loop

A self-loop counts for 2 in the degree function.

Proposition

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A complete graph K_n is a simple graph with all $n(n-1)/2$ possible edges, like the graph below for $n = 2, 3, 4, 5$.

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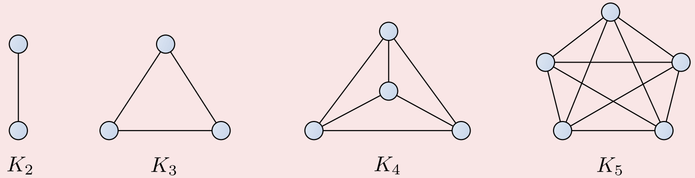

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We need to represent

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- Vertices

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First One

Adjacency Representation

Second One

Matrix Representation

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Basic Definition

It is an array of size $|V|$ with

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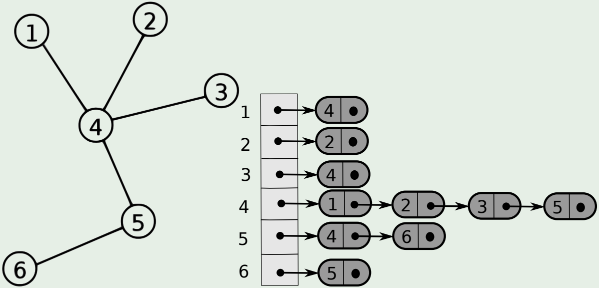

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Space for storage

For undirected or directed graphs $O\left(V+E\right)$

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Search: Successful or Unsuccessful

$O(1 + degree(v))$

In addition

Adjacency lists can readily be adapted to represent weighted graphs

- Weight function $w : E \to \mathbb{R}$
- The weight $w(u, v)$ of the edge $(u, v) \in E$ is simply stored with vertex v in u's adjacency list

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Possible Disadvantage

When looking to see if an edge exist

There is no quicker way to determine if a given edge (u,v)

Adjacency Matrix Representation

In a natural way the edges can be identified by the nodes

For example, the edge between 1 and 4 nodes gets named as (1,4)

Then

How, we use this to represent the graph through a Matrix or and Array of Arrays??!!!

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What about the following?

How do we indicate that an edge exist given the following matrix

$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & - & - & - & - & - & - \\
2 & - & - & - & - & - & - \\
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You say it!!

- Use a 0 for no-edge
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We have then...

Definition

- 0/1 $N \times N$ matrix with N =Number of nodes or vertices
- $A(i,j) = 1$ iff (i,j) is an edge

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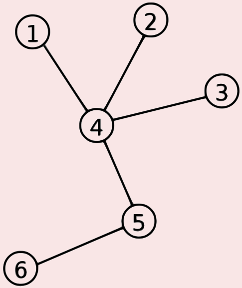
We have then...

For the previous example



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Properties of the Matrix for Undirected Graphs

Property One

Diagonal entries are zero.

Property Two

Adjacency matrix of an undirected graph is symmetric:

$$A(i,j) = A(j,i) \text{ for all } i \text{ and } j$$

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Complexity

Memory

$$\Theta\left(V^2\right) \tag{1}$$

Looking for an edge

$O(1)$

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Traversing the Graph

Why do we need to traverse the graph?

Do you have any examples?

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Breadth-first search

Definition

Given a graph $G = (V, E)$ and a source vertex s, breadth-first search systematically explores the edges of $G$to "discover" every vertex that is reachable from the vertex s

Something Notable

A vertex is discovered the first time it is **encountered** during the search

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Breadth-First Search Algorithm

Algorithm

BFS(G, s)

for each vertex $u \in G.V - \{s\}$
    $u.color =$ WHITE
    $u.d = \infty$
    $u.\pi = NIL$
$s.color =$GRAY
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$Q = \emptyset$
Enqueue(Q, s)
while $Q \neq \emptyset$
    $u =$Dequeue(Q)
    for each $v \in G.Adj[u]$
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BFS allows to change the order of recursion

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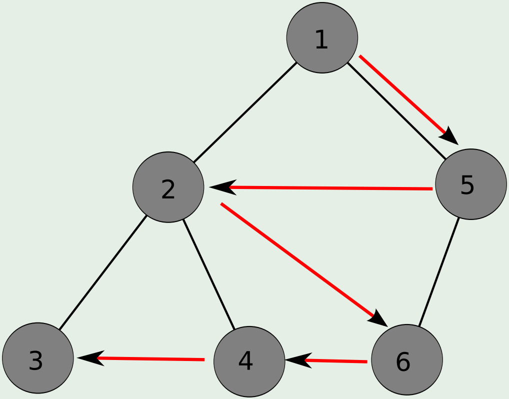

Loop Invariance

The While loop

This while loop maintains the following invariant :

- At the test in line 10, the queue Q consists of the set of gray vertices

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$Q = s$ and $s.color$ = GRAY

Maintenance

The inner loop only pushes gray nodes into the queue.

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Termination

When every node that can be visited is painted black

Example

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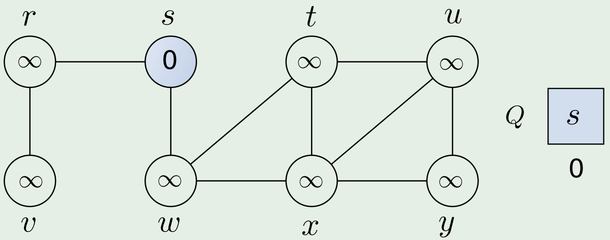

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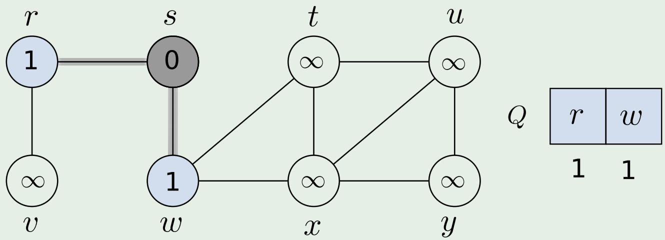

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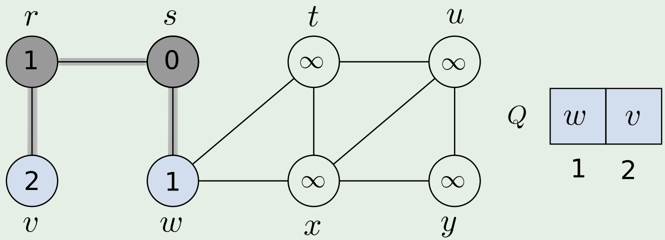

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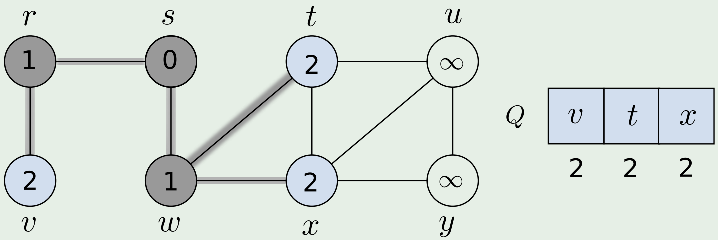

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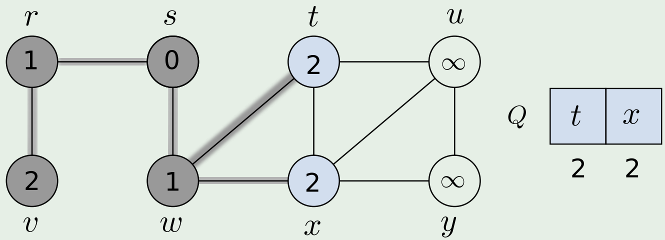

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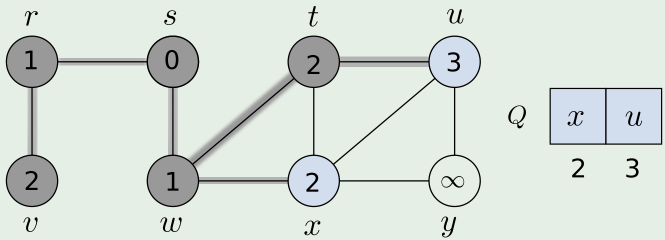

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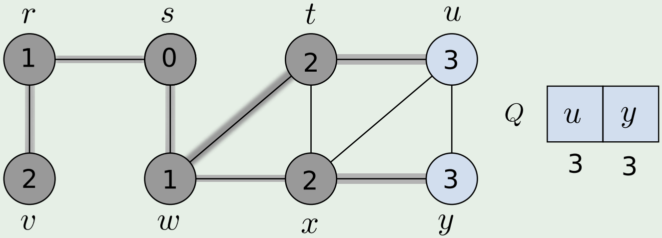

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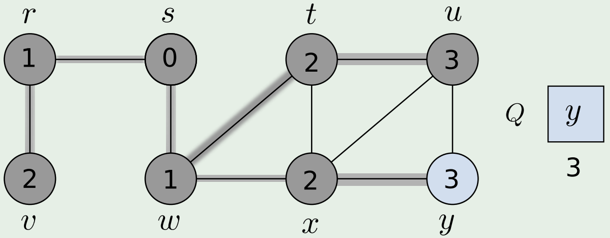

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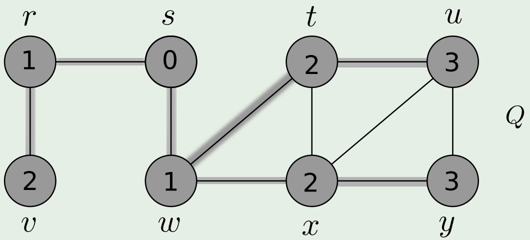

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What about the outer loop?

$O(V)$ Enqueue / Dequeue operations – Each adjacency list is processed only once.

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Properties: Predecessor Graph

Something Notable

Breadth-first search constructs a breadth-first tree, initially containing only its root, which is the source vertex s

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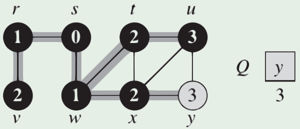

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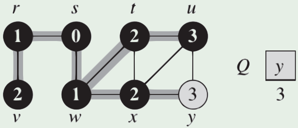

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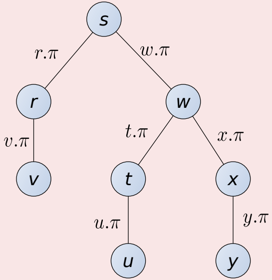

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Clearly

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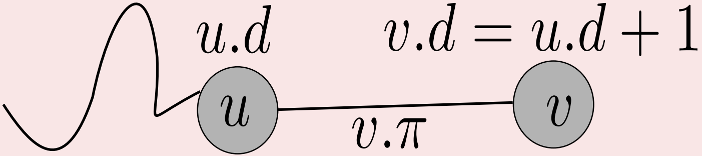
Distance Idea

- First, once a node u is reached from v, we use the previous shortest distance from s to update the node distance.



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You can use the idea of strong induction

- Given a branch of the breadth-first search $u\ s, u_1, u_2, ..., u_n$

$$u_n.\pi = u_{n-1}.\pi + 1 = u_{n-1}.\pi + 2 = n + s.\pi = n \tag{2}$$

Thus, we have that

- For any vertex $v \neq s$ that is reachable from s, one of the shortest path from s to v is
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Outline

1. Introduction
   - Graphs Everywhere
   - History
   - Basic Theory
   - Representing Graphs in a Computer
2. Traversing the Graph
   - Breadth-first search
   - Depth-First Search
3. Applications
   - Finding a path between nodes
   - Connected Components
   - Spanning Trees
   - Topological Sorting

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Depth-first search

Given G

- Pick an unvisited vertex v, remember the rest.
  - Recurse on vertices adjacent to v

The Pseudo-code

Code for DFS

DFS(G)

for each vertex $u \in G.V$

$\quad u.color =$ WHITE

$\quad u.\pi = NIL$

$time = 0$

for each vertex $u \in G.V$

$\quad$ **if** $u.color =$ WHITE

$\quad\quad$ **DFS-VISIT**(G, u)

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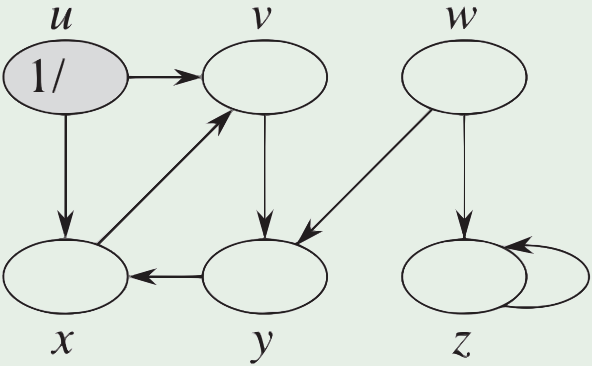

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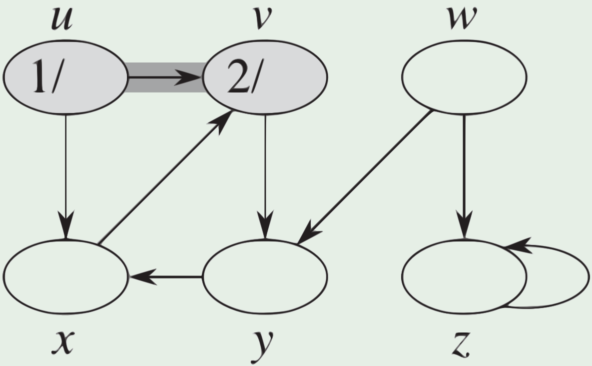

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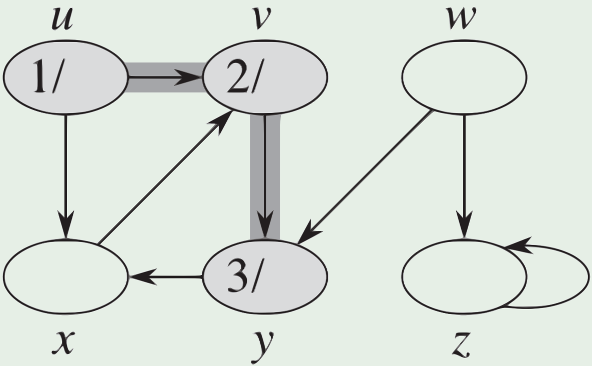

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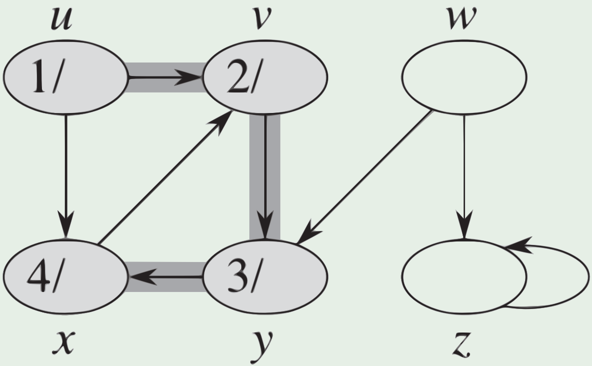

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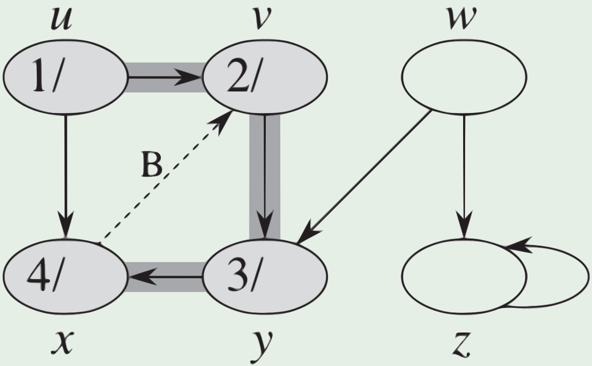

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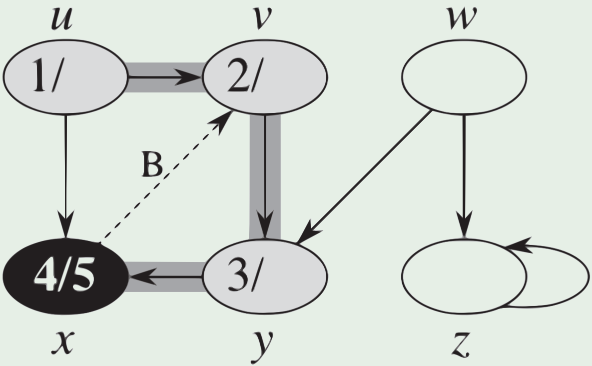

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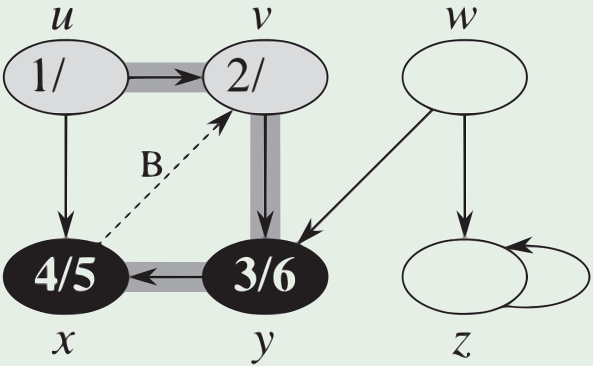

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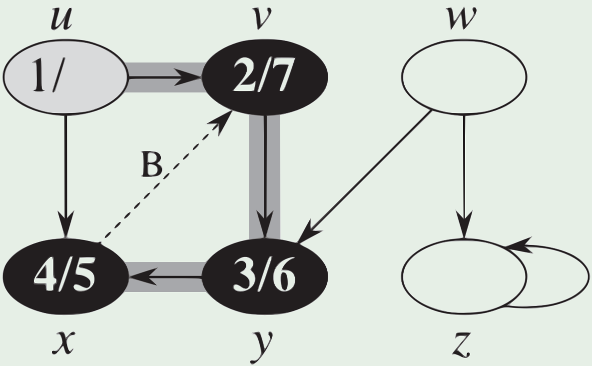

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1. The loops on lines 1–3 and lines 5–7 of DFS take $\Theta\left(V\right)$.
2. The procedure DFS-VISIT is called exactly once for each vertex $v \in V$.
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- Strongly Connected Components
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Finding a path between nodes

We do the following

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A connected component (or just component) of an undirected graph is a subgraph in which any two vertices are connected to each other by paths.

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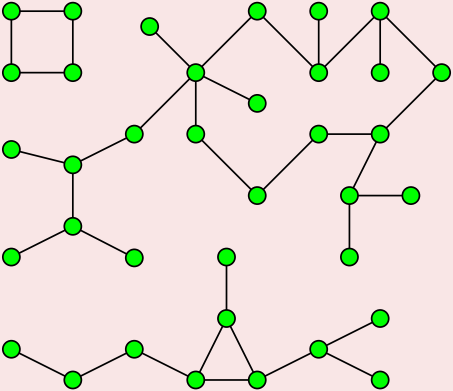

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Newly visited vertices (plus edges between them) define a component.

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Repeat until all vertices are visited.

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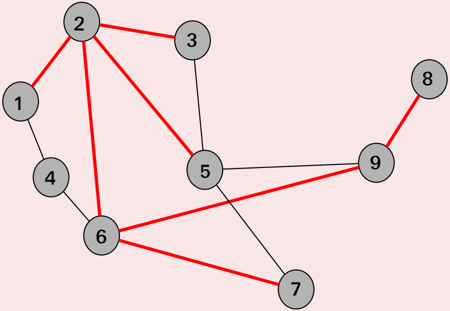

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Topological Sorting

Definitions

A topological sort (sometimes abbreviated topsort or toposort) or topological ordering of a directed graph is a linear ordering of its vertices such that for every directed edge (u, v) from vertex u to vertex y, u comes before v in the ordering.

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- The canonical application of topological sorting (topological order) is in scheduling a sequence of jobs or tasks based on their dependencies.

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- Topological sorting algorithms were first studied in the early 1960s in the context of the PERT technique for scheduling in project management (Jarnagin 1960).

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Then

We have that

The jobs are represented by vertices, and there is an edge from x to y if job x must be completed before job y can be started.

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When washing clothes, the washing machine must finish before we put the clothes to dry.

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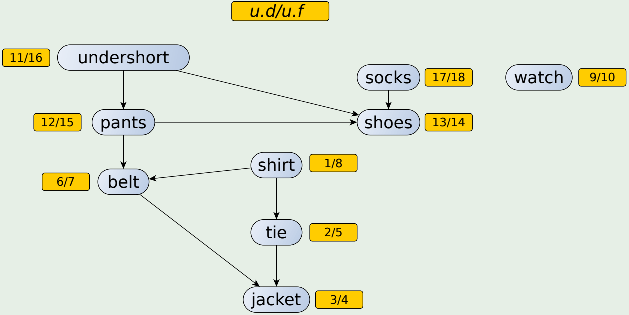

Thus

Using the $u.f$

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Example

After Sorting