Introduction to Artificial Intelligence

Multilayer Perceptron

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March 11, 2019

Outline

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Do you remember?

The Perceptron has the following problem

Given that the perceptron is a linear classifier

It is clear that

It will never be able to classify stuff that is not linearly separable

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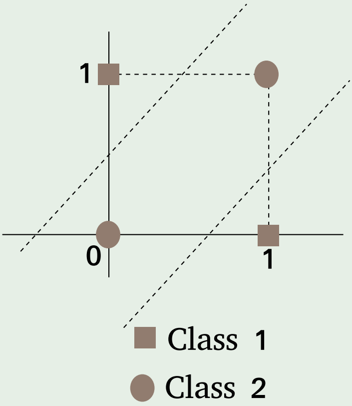
Example: XOR Problem

The Problem



The Perceptron cannot solve it

Because

The perceptron is a linear classifier!!!

Thus

Something needs to be done!!!

Maybe

Add an extra layer!!!

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A little bit of history

It was first cited by Vapnik

Vapnik cites (Bryson, A.E.; W.F. Denham; S.E. Dreyfus. Optimal programming problems with inequality constraints. I: Necessary conditions for extremal solutions. AIAA J. 1, 11 (1963) 2544-2550) as the first publication of the backpropagation algorithm in his book "Support Vector Machines."

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Arthur E. Bryson and Yu-Chi Ho described it as a multi-stage dynamic system optimization method in 1969.

However

It was not until 1974 and later, when applied in the context of neural networks and through the work of Paul Werbos, David E. Rumelhart, Geoffrey E. Hinton and Ronald J. Williams that it gained recognition.

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Then

Something Notable

It led to a "renaissance" in the field of artificial neural network research.

Nevertheless

During the 2000s it fell out of favour but has returned again in the 2010s, now able to train much larger networks using huge modern computing power such as GPUs.

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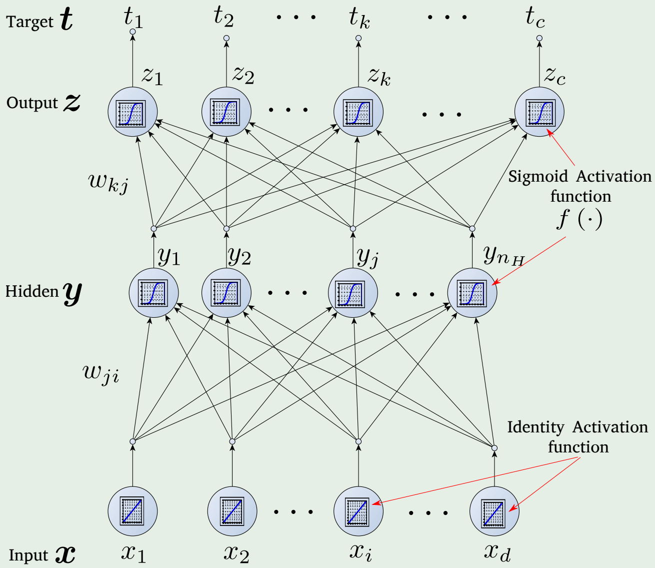
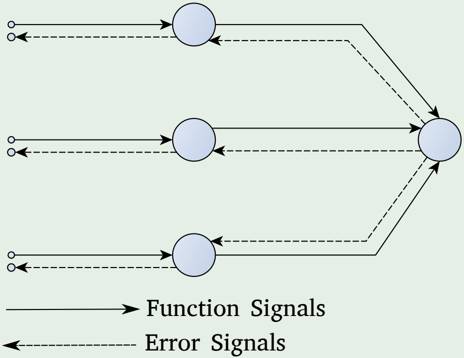
Multi-Layer Perceptron (MLP)

Multi-Layer Architecture.



Information Flow

We have the following information flow



Explanation

Problems with Hidden Layers

1. Increase complexity of Training
2. It is necessary to think about "Long and Narrow" network vs "Short and Fat" network.

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Intuition for a One Hidden Layer

1. For every input case of region, that region can be delimited by hyperplanes on all sides using hidden units on the first hidden layer.
2. A hidden unit in the second layer than ANDs them together to bound the region.

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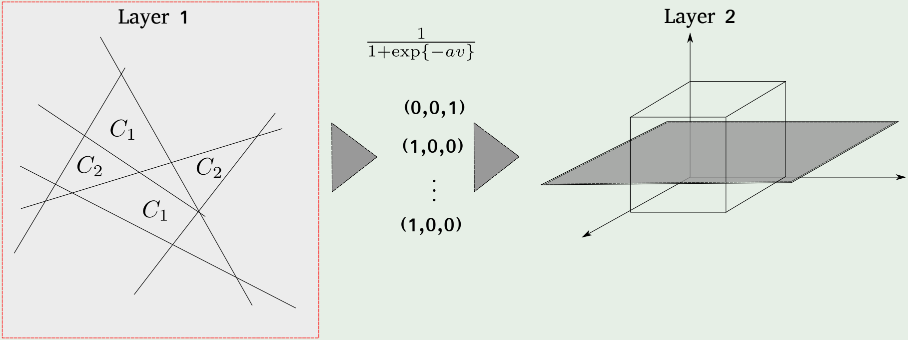

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Remember!!! The Quadratic Learning Error function

Cost Function our well know error at pattern m

$$J(m) = \frac{1}{2} e_k^2(m) \quad (1)$$

Delta Rule or Widrow-Hoff Rule

$$\Delta w_{kj}(m) = -\eta e_k(m) x_j(m) \quad (2)$$

Actually this is know as Gradient Descent

$$w_{kj}(m+1) = w_{kj}(m) + \Delta w_{kj}(m) \quad (3)$$

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Back-propagation

Setup

Let t_k be the k-th target (or desired) output and z_k be the k-th computed output with $k = 1, \dots, d$ and $\boldsymbol{w}$ represents all the weights of the network

Training Error for a single Pattern or Sample!!!

$$J(\boldsymbol{w}) = \frac{1}{2}\sum_{k=1}^{c}(t_k - z_k)^2 = \frac{1}{2}\|\boldsymbol{t} - \boldsymbol{z}\|^2 \quad (4)$$

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Gradient Descent

Gradient Descent

The back-propagation learning rule is based on gradient descent.

Reducing the Error

The weights are initialized with pseudo-random values and are changed in a direction that will reduce the error:

$$\Delta w = -\eta \frac{\partial J}{\partial w} \tag{5}$$

Where

η is the learning rate which indicates the relative size of the change in weights:

$$w(m+1) = w(m) + \Delta w(m) \tag{6}$$

where m is the m-th pattern presented

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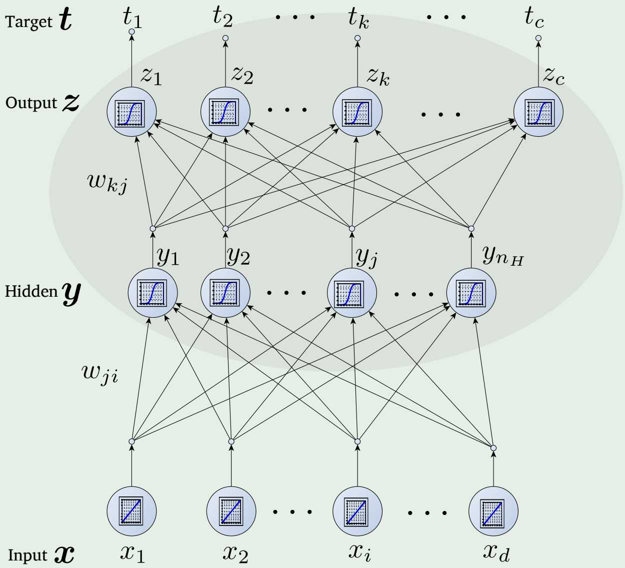
Multilayer Architecture

Multilayer Architecture: hidden–to-output weights



Observation about the activation function

Hidden Output is equal to

$$y_j = f\left(\sum_{i=1}^{d} w_{ji} x_i\right)$$

Output is equal to

$$z_k = f\left(\sum_{j=1}^{y_{n_H}} w_{kj} y_j\right)$$

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Hidden–to-Output Weights

Error on the hidden–to-output weights

$$\frac{\partial J}{\partial w_{kj}} = \frac{\partial J}{\partial net_k} \cdot \frac{\partial net_k}{\partial w_{kj}} = -\delta_k \cdot \frac{\partial net_k}{\partial w_{kj}} \quad (7)$$

net_k

It describes how the overall error changes with the activation of the unit's net:

$$net_k = \sum_{j=1}^{n_H} w_{kj} y_j = w_k^T \cdot y \quad (8)$$

Now

$$\delta_k = -\frac{\partial J}{\partial net_k} = -\frac{\partial J}{\partial z_k} \cdot \frac{\partial z_k}{\partial net_k} = (t_k - z_k) f'(net_k) \quad (9)$$

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Hidden–to-Output Weights

Why?

$$z_k = f\left(net_k\right) \quad (10)$$

Thus

$$\frac{\partial z_k}{\partial net_k} = f'\left(net_k\right) \quad (11)$$

Since $net_k = w_k^T \cdot y$ therefore:

$$\frac{\partial net_k}{\partial w_{kj}} = y_j \quad (12)$$

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Finally

The weight update (or learning rule) for the hidden-to-output weights is:

$$\triangle w_{kj} = \eta \delta_k y_j = \eta \left(t_k - z_k \right) f' \left(net_k \right) y_j \tag{13}$$

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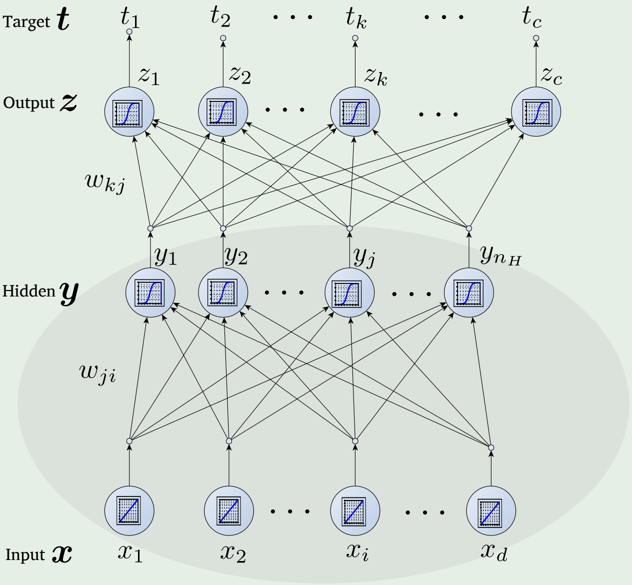
Multi-Layer Architecture

Multi-Layer Architecture: Input–to-Hidden weights



Input–to-Hidden Weights

Error on the Input–to-Hidden weights

$$\frac{\partial J}{\partial w_{ji}} = \frac{\partial J}{\partial y_j} \cdot \frac{\partial y_j}{\partial net_j} \cdot \frac{\partial net_j}{\partial w_{ji}} \tag{14}$$

Thus

$$\begin{aligned}
\frac{\partial J}{\partial y_j} &= \frac{\partial}{\partial y_j}\left[\frac{1}{2}\sum_{k=1}^{c}\left(t_k - z_k\right)^2\right] \\
&= -\sum_{k=1}^{c}\left(t_k - z_k\right)\frac{\partial z_k}{\partial y_j} \\
&= -\sum_{k=1}^{c}\left(t_k - z_k\right)\frac{\partial z_k}{\partial net_k} \cdot \frac{\partial net_k}{\partial y_j} \\
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&= -\sum_{k=1}^{c}(t_k - z_k)\frac{\partial f(net_k)}{\partial net_k} \cdot w_{kj}
\end{aligned}$$

Input–to-Hidden Weights

Finally

$$\frac{\partial J}{\partial y_j} = -\sum_{k=1}^{c} (t_k - z_k) f'(net_k) \cdot w_{kj} \quad (15)$$

Remember

$$\delta_k = -\frac{\partial J}{\partial net_k} = (t_k - z_k) f'(net_k) \quad (16)$$

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What is $\frac{\partial y_j}{\partial net_j}$?

First

$$net_j = \sum_{i=1}^{d} w_{ji} x_i = \boldsymbol{w}_j^T \cdot \boldsymbol{x} \tag{17}$$

Then

$$y_j = f\left(net_j\right)$$

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Then, we can define δ_j

By defying the sensitivity for a hidden unit:

$$\delta_j = f'(net_j) \sum_{k=1}^{c} w_{kj}\delta_k \tag{18}$$

Which means that:

"The sensitivity at a hidden unit is simply the sum of the individual sensitivities at the output units weighted by the **hidden-to-output weights** w_{kj}; all multiplied by $f'(net_j)$"

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What about $\frac{\partial net_j}{\partial w_{ji}}$?

We have that

$$\frac{\partial net_j}{\partial w_{ji}} = \frac{\partial \boldsymbol{w}_j^T \cdot \boldsymbol{x}}{\partial w_{ji}} = \frac{\partial \sum_{i=1}^{d} w_{ji} x_i}{\partial w_{ji}} = x_i$$

Finally

The learning rule for the input-to-hidden weights is:

$$\Delta w_{ji} = \eta x_i \delta_j = \eta \left[\sum_{k=1}^{c} w_{kj} \delta_k \right] f'\left(net_j\right) x_i \qquad (19)$$

Basically, the entire training process has the following steps

Initialization

Assuming that no prior information is available, pick the synaptic weights and thresholds

Forward Computation

Compute the induced function signals of the network by proceeding forward through the network, layer by layer.

Backward Computation

Compute the local gradients of the network.

Finally

Adjust the weights!!!

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Now, Calculating Total Change

We have for that

The Total Training Error is the sum over the errors of N individual patterns

The Total Training Error

$$J = \sum_{p=1}^{N} J_p = \frac{1}{2} \sum_{p=1}^{N} \sum_{k=1}^{d} \left(t_k^p - z_k^p\right)^2 = \frac{1}{2} \sum_{p=1}^{n} \left\| t^p - z^p \right\|^2 \quad (20)$$

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About the Total Training Error

Remarks

- A weight update may reduce the error on the single pattern being presented but can increase the error on the full training set.
- However, given a large number of such individual updates, the total error of equation (20) decreases.

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A simple way to stop the training

- The algorithm terminates when the change in the criterion function $J(\boldsymbol{w})$ is smaller than some preset value Θ.

$$\Delta J(\boldsymbol{w}) = |J(\boldsymbol{w}(t+1)) - J(\boldsymbol{w}(t))| \tag{21}$$

- There are other stopping criteria that lead to better performance than this one.

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Other Stopping Criteria

Norm of the Gradient

The back-propagation algorithm is considered to have converged when the Euclidean norm of the gradient vector reaches a sufficiently small gradient threshold.

$$\left\|\nabla_{\boldsymbol{w}} J\left(m\right)\right\| < \Theta \tag{22}$$

Rate of change in the average error per epoch

The back-propagation algorithm is considered to have converged when the absolute rate of change in the average squared error per epoch is sufficiently small.

$$\left|\frac{1}{N}\sum_{p=1}^{N} J_p\right| < \Theta \tag{23}$$

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About the Stopping Criteria

Observations

1. Before training starts, the error on the training set is high.
    - Through the learning process, the error becomes smaller.
2. The error per pattern depends on the amount of training data and the expressive power (such as the number of weights) in the network.
3. The average error on an independent test set is always higher than on the training set, and it can decrease as well as increase.
4. A validation set is used in order to decide when to stop training.
    - We do not want to over-fit the network and decrease the power of the classifier generalization "we stop training at a minimum of the error on the validation set"

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3. The average error on an independent test set is always higher than on the training set, and it can decrease as well as increase.
4. A validation set is used in order to decide when to stop training.
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Observations

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Epoch

As with other types of backpropagation, 'learning' is a supervised process that occurs with each cycle or 'epoch' through a forward activation flow of outputs, and the backwards error propagation of weight adjustments.

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Outline

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Final Basic Batch Algorithm

Perceptron($\boldsymbol{X}$)

Initialize random w, **number of hidden units** n_H, **number of outputs** z, **stopping criterion** Θ, **learning rate** η, **epoch** $m = 0$
do
  $m = m + 1$
    for $s = 1$ **to** N
      $x(m) = X(:, s)$
      for $k = 1$ **to** c
        $\delta_k = (t_k - z_k) f'\left(w_k^T \cdot y\right)$
        for $j = 1$ **to** n_H
          $net_j = w_j^T \cdot x; y_j = f\left(net_j\right)$
          $w_{kj}(m) = w_{kj}(m) + \eta \delta_k y_j(m)$
      for $j = 1$ **to** n_H
        $\delta_j = f'\left(net_j\right) \sum_{k=1}^{c} w_{kj} \delta_k$
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          $w_{ji}(m) = w_{ji}(m) + \eta \delta_j x_i(m)$
until $\|\nabla_w J(m)\| < \Theta$
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5. $\qquad\quad \boldsymbol{x}(m) = \boldsymbol{X}(:, s)$
6. $\qquad\quad$ **for** $k = 1$ **to** c
7. $\qquad\qquad \delta_k = (t_k - z_k)\, f'\left(\boldsymbol{w}_k^T \cdot \boldsymbol{y}\right)$
8. $\qquad\qquad$ **for** $j = 1$ **to** n_H
9. $\qquad\qquad\quad net_j = \boldsymbol{w}_j^T \cdot \boldsymbol{x}; y_j = f\left(net_j\right)$
10. $\qquad\qquad\quad w_{kj}(m) = w_{kj}(m) + \eta \delta_k y_j(m)$
11. $\qquad\quad$ **for** $j = 1$ **to** n_H
12. $\qquad\qquad \delta_j = f'\left(net_j\right) \sum_{k=1}^{c} w_{kj} \delta_k$
13. $\qquad\qquad$ **for** $i = 1$ **to** d
14. $\qquad\qquad\quad w_{ji}(m) = w_{ji}(m) + \eta \delta_j x_i(m)$
15. **until** $\|\nabla_{\boldsymbol{w}} J(m)\| < \Theta$
16. **return** $\boldsymbol{w}(m)$

Outline

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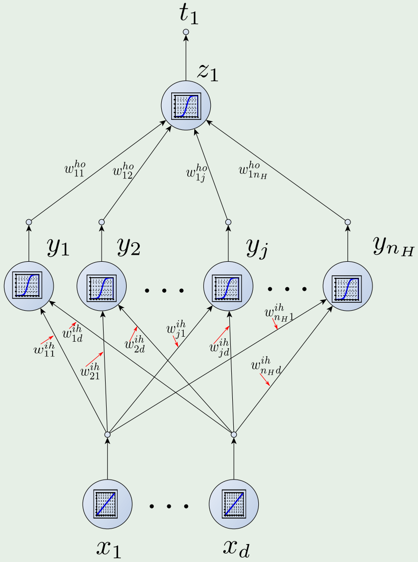
Example of Architecture to be used

Given the following Architecture and assuming N samples



Outline

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Generating the output z_k

Given the input

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 & \cdots & \boldsymbol{x}_N \end{bmatrix} \tag{24}$$

Where

x_i is a vector of features

$$x_i = \begin{pmatrix} x_{1i} \\ x_{2i} \\ \vdots \\ x_{di} \end{pmatrix} \tag{25}$$

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Therefore

We must have the following matrix for the input to hidden inputs

$$\boldsymbol{W}_{IH} = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1d} \\ w_{21} & w_{22} & \cdots & w_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_H 1} & w_{n_H 2} & \cdots & w_{n_H d} \end{pmatrix} = \begin{pmatrix} \boldsymbol{w}_1^T \\ \boldsymbol{w}_2^T \\ \vdots \\ \boldsymbol{w}_{n_H}^T \end{pmatrix} \qquad (26)$$

Given that $\boldsymbol{w}_j = \begin{pmatrix} w_{j1} \\ w_{j2} \\ \vdots \\ w_{jd} \end{pmatrix}$

Thus

We can create the net_j for all the inputs by simply

$$net_j = \boldsymbol{W}_{IH}\boldsymbol{X} = \begin{pmatrix} \boldsymbol{w}_1^T\boldsymbol{x}_1 & \boldsymbol{w}_1^T\boldsymbol{x}_2 & \cdots & \boldsymbol{w}_1^T\boldsymbol{x}_N \\ \boldsymbol{w}_2^T\boldsymbol{x}_1 & \boldsymbol{w}_2^T\boldsymbol{x}_2 & \cdots & \boldsymbol{w}_2^T\boldsymbol{x}_N \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{w}_{n_H}^T\boldsymbol{x}_1 & \boldsymbol{w}_{n_H}^T\boldsymbol{x}_2 & \cdots & \boldsymbol{w}_{n_H}^T\boldsymbol{x}_N \end{pmatrix} \qquad (27)$$

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$$

Now, we need to generate the $\boldsymbol{y}_k$

We apply the activation function element by element in $\boldsymbol{net}_j$

$$\boldsymbol{y}_1 = \begin{pmatrix} f\left(\boldsymbol{w}_1^T \boldsymbol{x}_1\right) & f\left(\boldsymbol{w}_1^T \boldsymbol{x}_2\right) & \cdots & f\left(\boldsymbol{w}_1^T \boldsymbol{x}_N\right) \\ f\left(\boldsymbol{w}_2^T \boldsymbol{x}_1\right) & f\left(\boldsymbol{w}_2^T \boldsymbol{x}_2\right) & \cdots & f\left(\boldsymbol{w}_2^T \boldsymbol{x}_N\right) \\ \vdots & \vdots & \ddots & \vdots \\ f\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_1\right) & f\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_2\right) & \cdots & f\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_N\right) \end{pmatrix} \quad (28)$$

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IMPORTANT about overflows!!!

- Be careful about the numeric stability of the activation function.
- I the case of python, we can use the ones provided by scipy.special

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However, We can create a Sigmoid function

It is possible to use the following pseudo-code

Sigmoid(x)

```
if x < -BIGREAL
    return 0
else if x > BIGREAL
    return 1
else
    return 1.0 / (1.0 + exp{-αx})  ◁ 1.0 refers to the floating point (Rationals
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```

Outline

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For this, we get $\boldsymbol{net}_k$

For this, we obtain the $\boldsymbol{W}_{HO}$

$$\boldsymbol{W}_{HO} = \begin{pmatrix} w^o_{11} & w^o_{12} & \cdots & w^o_{1n_H} \end{pmatrix} = \begin{pmatrix} \boldsymbol{w}_o^T \end{pmatrix} \tag{29}$$

Thus

$$\boldsymbol{net}_k = \begin{pmatrix} w^o_{11} & w^o_{12} & \cdots & w^o_{1n_H} \end{pmatrix} \begin{pmatrix} f\left(\boldsymbol{w}_1^T \boldsymbol{x}_1\right) & f\left(\boldsymbol{w}_1^T \boldsymbol{x}_2\right) & \cdots & f\left(\boldsymbol{w}_1^T \boldsymbol{x}_N\right) \\ f\left(\boldsymbol{w}_2^T \boldsymbol{x}_1\right) & f\left(\boldsymbol{w}_2^T \boldsymbol{x}_2\right) & \cdots & f\left(\boldsymbol{w}_2^T \boldsymbol{x}_N\right) \\ \vdots & \vdots & \ddots & \vdots \\ \underbrace{f\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_1\right)}_{\boldsymbol{y}_{k1}} & \underbrace{f\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_2\right)}_{\boldsymbol{y}_{k2}} & \cdots & \underbrace{f\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_N\right)}_{\boldsymbol{y}_{kN}} \end{pmatrix} \tag{30}$$

In matrix notation

$$\boldsymbol{net}_k = \begin{pmatrix} \boldsymbol{w}_o^T \boldsymbol{y}_{k1} & \boldsymbol{w}_o^T \boldsymbol{y}_{k2} & \cdots & \boldsymbol{w}_o^T \boldsymbol{y}_{kN} \end{pmatrix} \tag{31}$$

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Outline

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Now, we have

Thus, we have $\boldsymbol{z}_k$ (In our case $k = 1$, but it could be a range of values)

$$\boldsymbol{z}_k = \begin{pmatrix} f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1}\right) & f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k2}\right) & \cdots & f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN}\right) \end{pmatrix} \quad (32)$$

Thus, we generate a vector of differences

$$\boldsymbol{d} = \boldsymbol{t} - \boldsymbol{z}_k = \begin{pmatrix} t_1 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1}\right) & t_2 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k2}\right) & \cdots & t_N - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN}\right) \end{pmatrix} \quad (33)$$

where $\boldsymbol{t} = \begin{pmatrix} t_1 & t_2 & \cdots & t_N \end{pmatrix}$ is a row vector of desired outputs for each sample.

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Now, we multiply element wise

We have the following vector of derivatives of net

$$\boldsymbol{D}_f = \begin{pmatrix} \eta f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1}\right) & \eta f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k2}\right) & \cdots & \eta f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN}\right) \end{pmatrix} \quad (34)$$

where η is the step rate.

Finally, by element wise multiplication (Hadamard Product)

$$\boldsymbol{d} = \big(\eta\left[t_1 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1}\right)\right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1}\right) \quad \eta\left[t_2 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k2}\right)\right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k2}\right) \quad \cdots \quad \eta\left[t_N - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN}\right)\right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN}\right) \big)$$

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Tile $\boldsymbol{d}$

Tile downward

$$\boldsymbol{d}_{tile} = n_H \text{ rows } \left\{ \begin{pmatrix} \boldsymbol{d} \\ \boldsymbol{d} \\ \vdots \\ \boldsymbol{d} \end{pmatrix} \right. \tag{35}$$

Finally, we multiply element wise against $\boldsymbol{y}_1$ (Hadamard Product)

$$\Delta w_{1j}^{temp} = \boldsymbol{y}_1 \circ \boldsymbol{d}_{tile} \tag{36}$$

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Finally, we multiply element wise against $\boldsymbol{y}_1$ (Hadamard Product)

$$\Delta \boldsymbol{w}_{1j}^{temp} = \boldsymbol{y}_1 \circ \boldsymbol{d}_{tile} \tag{36}$$

We obtain the total $\Delta \boldsymbol{w}_{1j}$

We sum along the rows of $\Delta \boldsymbol{w}_{1j}^{temp}$

$$\Delta \boldsymbol{w}_{1j} = \begin{pmatrix} \eta \left[t_1 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1} \right) \right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1} \right) y_{11} + \eta \left[t_1 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN} \right) \right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN} \right) y_{1N} \\ \vdots \\ \eta \left[t_1 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1} \right) \right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{k1} \right) y_{n_H 1} + \eta \left[t_1 - f\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN} \right) \right] f'\left(\boldsymbol{w}_o^T \boldsymbol{y}_{kN} \right) y_{n_H N} \end{pmatrix} \tag{37}$$

where $y_{hm} = f\left(\boldsymbol{w}_h^T \boldsymbol{x}_m \right)$ with $h = 1, 2, ..., n_H$ and $m = 1, 2, ..., N$.

Finally, we update the first weights

We have then

$$\boldsymbol{W}_{HO}\left(t+1\right)=\boldsymbol{W}_{HO}\left(t\right)+\Delta\boldsymbol{w}_{1j}^{T}\left(t\right) \tag{38}$$

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Outline

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First

We multiply element wise the $\boldsymbol{W}_{HO}$ and $\Delta \boldsymbol{w}_{1j}$

$$\boldsymbol{T} = \Delta \boldsymbol{w}_{1j}^{T} \circ \boldsymbol{W}_{HO}^{T} \tag{39}$$

Now, we obtain the element wise derivative of net_j

$$\boldsymbol{Dnet}_j = \begin{pmatrix} f'\left(\boldsymbol{w}_1^T \boldsymbol{x}_1\right) & f'\left(\boldsymbol{w}_1^T \boldsymbol{x}_2\right) & \cdots & f'\left(\boldsymbol{w}_1^T \boldsymbol{x}_N\right) \\ f'\left(\boldsymbol{w}_2^T \boldsymbol{x}_1\right) & f'\left(\boldsymbol{w}_2^T \boldsymbol{x}_2\right) & \cdots & f'\left(\boldsymbol{w}_2^T \boldsymbol{x}_N\right) \\ \vdots & \vdots & \ddots & \vdots \\ f'\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_1\right) & f'\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_2\right) & \cdots & f'\left(\boldsymbol{w}_{n_H}^T \boldsymbol{x}_N\right) \end{pmatrix} \tag{40}$$

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Thus

We tile to the right T

$$\boldsymbol{T}_{tile} = \underbrace{\begin{pmatrix} \boldsymbol{T} & \boldsymbol{T} & \cdots & \boldsymbol{T} \end{pmatrix}}_{N \text{ Columns}} \tag{41}$$

Now, we multiply element wise together with η

$$\boldsymbol{P}_t = \eta \left(\boldsymbol{Dnet}_j \circ \boldsymbol{T}_{tile} \right) \tag{42}$$

where η is constant multiplied against the result the Hadamar Product (Result a $n_H \times N$ matrix)

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Finally

We get use the transpose of $\boldsymbol{X}$ which is a $N \times d$ matrix

$$\boldsymbol{X}^T = \begin{pmatrix} \boldsymbol{x}_1^T \\ \boldsymbol{x}_2^T \\ \vdots \\ \boldsymbol{x}_N^T \end{pmatrix} \quad (43)$$

Finally, we get a $n_H \times d$ matrix

$$\Delta w_{ij} = \boldsymbol{P}_i \boldsymbol{X}^T \quad (44)$$

Thus, given $\boldsymbol{W}_{IH}$

$$\boldsymbol{W}_{IH}(t+1) = \boldsymbol{W}_{HO}(t) + \Delta w_{ij}^T(t) \quad (45)$$

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Outline

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We have different activation functions

The two most important

1. Sigmoid function.
2. Hyperbolic tangent function

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Logistic Function

This non-linear function has the following definition for a neuron j

$$f_j\left(v_j\left(n\right)\right) = \frac{1}{1 + \exp\left\{-a v_j\left(n\right)\right\}} \; a > 0 \text{ and } -\infty < v_j\left(n\right) < \infty \tag{46}$$

Example

Logistic Function

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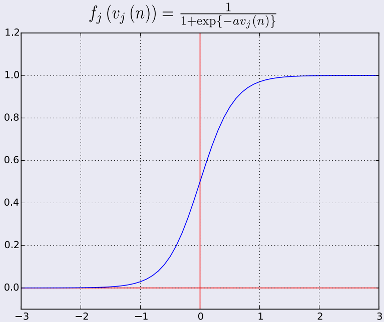
Example



The differential of the sigmoid function

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The outputs finish as

For the output neurons

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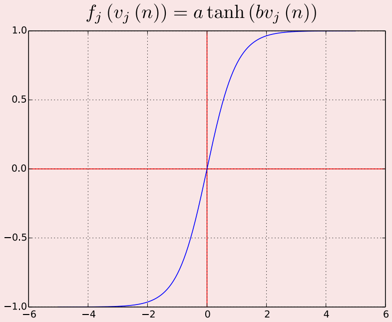

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Two ways of achieving this, LeCun 1993

- The use of an example that results in the largest training error.
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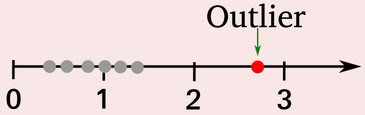

Outline

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Activation Function

We say that

An activation function $f(v)$ is antisymmetric if $f(-v) = -f(v)$

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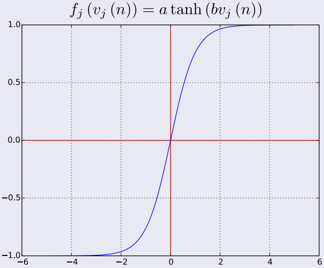

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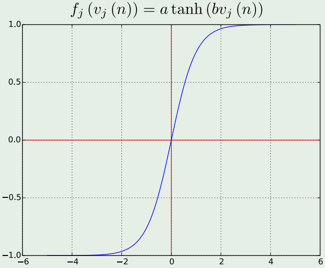

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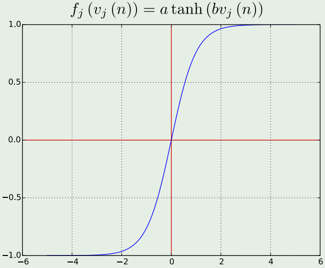

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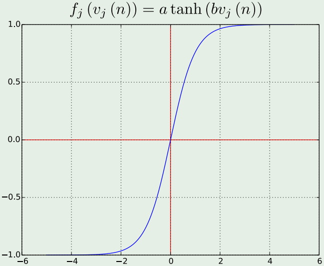

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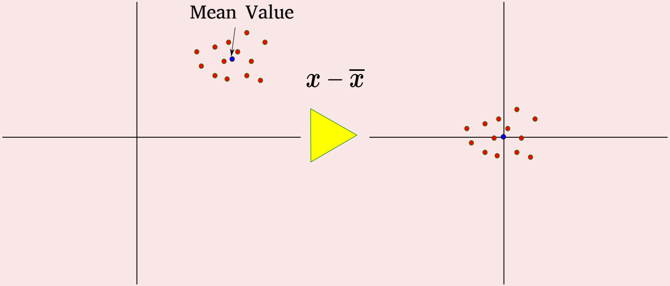

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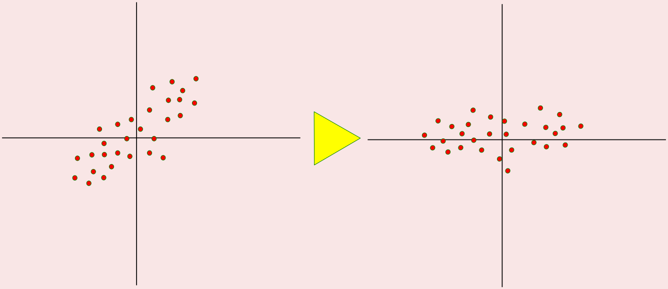

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Why this is advocated in Artificial Neural Networks

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Artificial neural networks that perform local computations are often held up as metaphors for biological neural networks.

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The use of local computations permits a graceful degradation in performance due to hardware errors, and therefore provides the basis for a fault-tolerant network design.

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Something Notable

The computational complexity of an algorithm is usually measured in terms of the number of multiplications, additions, and storage involved in its implementation.

- This is the electrical engineering approach!!!

Taking in account the total number of synapses, W including biases

We have $\triangle w_{kj} = \eta \delta_k y_j = \eta (t_k - z_k) f'(net_k) y_j$ (Backward Pass)

We have that for this step

1. We need to calculate net_k linear in the number of weights.
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The Complexity of the multi-layer perceptron is

$$O(W) \text{ Complexity}$$

Exercises

We have from NN by Haykin

4.2, 4.3, 4.6, 4.8, 4.16, 4.17, 3.7