Introduction to Artificial Intelligence

Introduction to Bayesian Networks

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Outline

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History

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- '60s The first expert systems. IF-THEN rules.
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- 1973 Gave up - to heavy calculations! (Gorry).
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Bayesian Networks are use in

- Spam Detection.
- Gene Discovery.
- Signal Processing.
- Ranking.
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Something Notable

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Bayesian Network Advantages

Many of Them

1. Since in a Bayesian network encodes all variables, missing data entries can be handled successfully.
2. When used for learning casual relationships, they help better understand a problem domain as well as forecast consequences.
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Bayes Theorem

One Version

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- $P(A|B)$ is the conditional probability of A, given B. It is also called the posterior probability because it is derived from or depends upon the specified value of B.
- $P(B|A)$ is the conditional probability of B given A. It is also called the likelihood.
- $P(B)$ is the prior or marginal probability of B, and acts as a normalizing constant.

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- $P(A)$ is the **prior probability** or marginal probability of A. It is "prior" in the sense that it does not take into account any information about B.
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What is the probability that a person has taken the drug?

$$P\left(D=y|T=+ve\right)=\frac{P\left(T=+ve|D=y\right)P\left(D{=}\text{y}\right)}{P\left(T=+ve|D=y\right)P\left(D{=}\text{y}\right)+P\left(T=+ve|D=n\right)P\left(D{=}\text{n}\right)}$$

Let me develop the equation

Using simply

$$P\left(A,B\right)=P\left(A|B\right)P\left(B\right)\ \text{(Chain Rule)} \tag{1}$$

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A More Complex Case

Increase Complexity

- Suppose now that there is a similar link between Lung Cancer (L) and a chest X-ray (X) and that we also have the following relationships:
  - History of smoking (S) has a direct influence on bronchitis (B) and lung cancer (L);
  - L and B have a direct influence on fatigue (F).

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Short Hand

$$P\left(b_1|s_1,f_1,x_1\right)=\frac{P\left(b_1,s_1,f_1,x_1\right)}{P\left(s_1,f_1,x_1\right)}=\frac{\sum_l P\left(b_1,s_1,f_1,x_1,l\right)}{\sum_{b,l} P\left(b,s_1,f_1,x_1,l\right)}$$

Values for the Complex Case

Table

Feature	Value	When the Feature Takes this Value
H	h_1	There is a history of smoking
	h_2	There is no history of smoking
B	b_1	Bronchitis is present
	b_2	Bronchitis is absent
L	l_1	Lung cancer is present
	l_2	Lung cancer is absent
F	f_1	Fatigue is present
	f_2	Fatigue is absent
C	c_1	Chest X-ray is positive
	c_2	Chest X-ray is negative

Problem with Large Instances

The joint probability distribution $P(H, B, L, F, C)$

- For five binary variables there are $2^5 = 32$ values in the joint distribution (for 100 variables there are over 2^{100} values)
- How are these values to be obtained?

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A Bayesian network consists of

- A Graph
  - Nodes represent the random variables.
  - Directed edges (arrows) between pairs of nodes.
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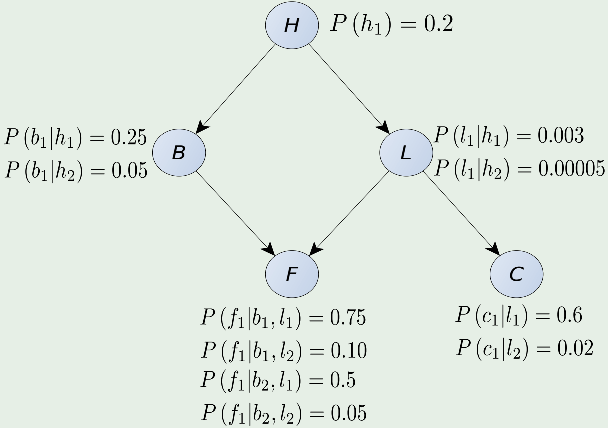
Example

DAG for the previous Lung Cancer Problem



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Markov Condition

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- Suppose we have a joint probability distribution P of the random variables in some set V and a DAG $G = (V, E)$.
  - We say that (G, P) satisfies **the Markov condition** if for each variable $X \in V$, $\{X\}$ is conditionally independent of the set of all its non-descendents given the set of all its parents.

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- $\text{PA}_X =$ set of parents of X.
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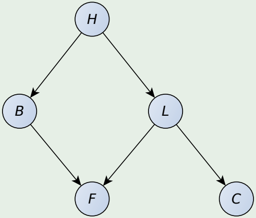

Given the previous DAG we have

Node	PA	Conditional Independence
C	$\{L\}$	$I_P(\{C\}, \{H, B, F\} \mid \{L\})$
B	$\{H\}$	$I_P(\{B\}, \{L, C\} \mid \{H\})$
F	$\{B, L\}$	$I_P(\{F\}, \{H, C\} \mid \{B, L\})$
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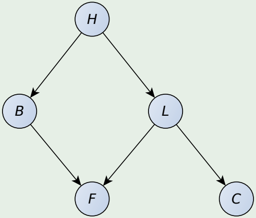

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Using the Markov Condition

First Decompose a Joint Distribution using the Chain Rule

$$P(c, f, l, b, h) = P(c|b, s, l, f)\, P(f|b, h, l)\, P(l|b, h)\, P(b|h)\, P(h) \quad (2)$$

Using the Markov condition in the following DAG

We have the following equivalences

- $P(c|b, h, l, f) = P(c|l)$
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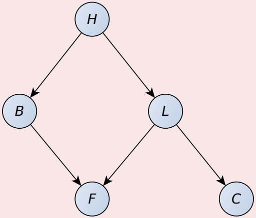

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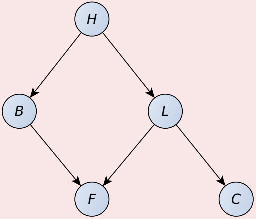

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Using the Markov Condition

Finally

$$P(c, f, l, b, h) = P(c|l)\, P(f|b, l)\, P(l|h)\, P(b|h)\, P(h) \quad (3)$$

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Representing the Joint Distribution

Theorem (Product of Conditional Probabilities of the Parents)

If (G, P) satisfies the Markov condition, then P is equal to the product of its conditional distributions of all nodes given values of their parents, whenever these conditional distributions exist.

General Representation

- In general, for a network with nodes $X_1, X_2, ..., X_n \Rightarrow$

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Proof

We prove the case where P is discrete

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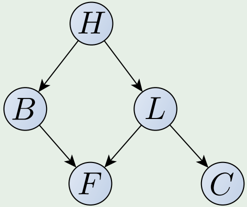

The ancestral ordering are

$$[H, L, B, C, F] \text{ and } [H, B, L, F, C] \tag{4}$$

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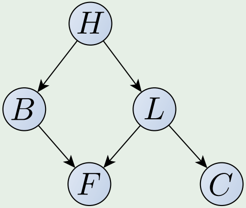

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Proof

Now

Let $X_1, X_2, ..., X_n$ be the resultant ordering.

For a given set of values of $x_1, x_2, ..., x_n$

Let pa_i be the subsets of these values containing the values of $X_i's$ parents

Thus, we need to prove that whenever $P(\text{pa}_i) \neq 0$ for $1 \leq i \leq n$

$$P(x_n, x_{n-1}, ..., x_1) = P(x_n|\text{pa}_n)\,P(x_{n-1}|\text{pa}_{n-1})\ldots P(x_1|\text{pa}_1) \quad (5)$$

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Proof

Something Notable

We show this using induction on the number of variables in the network.

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$$P(x_1) = P(x_1|\mathrm{pa}_1) \quad (6)$$

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Case 1

For this combination of values:

$$P\left(x_i, x_{i-1}, ..., x_1\right) = 0 \tag{9}$$

By Conditional Probability, we have

$$P\left(x_{i+1}, x_i, ..., x_1\right) = P\left(x_{i+1}|x_i, ..., x_1\right) P\left(x_i, ..., x_1\right) = 0 \tag{10}$$

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Proof

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Proof

Due to the previous equalities and the inductive hypothesis

There is some k, $1 \leq k \leq i$ such that $P\left(x_k|\text{pa}_k\right) = 0$ because after all

$$P\left(x_i|\text{pa}_i\right) P\left(x_{i-1}|\text{pa}_{i-1}\right) ... P\left(x_1|\text{pa}_1\right) = 0 \quad (11)$$

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- Suppose we have a joint probability distribution P of the random variables in some set V and a DAG $G = (V, E)$.
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Proof

Given this Markov Condition and the fact that $X_1, ..., X_i$ are all non-descendants of X_{i+1}

We have that

$$P\left(x_{i+1}, x_i, ..., x_1\right) = P\left(x_{i+1}|\text{pa}_{i+1}\right) P\left(x_i, ..., x_1\right)$$
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Q.E.D.

Outline

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Example

Imagine the following Random Variables

Variable	Value	Outcome
V	v_1	All objects containing a '1'
	v_2	All objects containing a '2'
S	s_1	All square objects
	s_2	All round objects
C	c_1	All black objects
	c_2	All white objects

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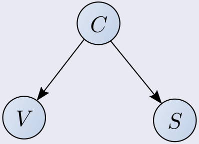
Example

Using the following graph



Using the chain rule

$$
\begin{aligned}
P(v, s, c) &= P(v|s, c)\, P(s|c)\, P(c) \\
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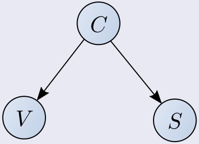

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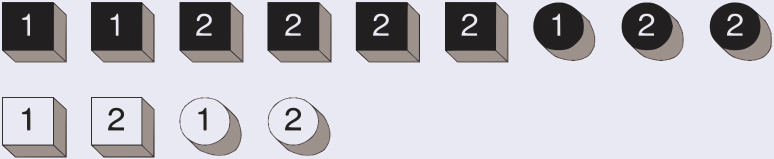
We have



c	s	v	$P(v\|s,c)$	$P(v\|c)$
c_1	s_1	v_1	1/3	1/3
c_1	s_1	v_2	2/3	2/3
c_1	s_2	v_1	1/3	1/3
c_1	s_2	v_2	2/3	2/3
c_2	s_1	v_1	1/2	1/2
c_2	s_1	v_2	1/2	1/2
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Therefore

We have the following probabilities

$$P\left(c_1, s_1, v_1\right) = \frac{2}{13}$$

With the following using the Markov Condition

$$P\left(v_1|c_1\right) P\left(s_1|c_1\right) P\left(c_1\right) = P\left(One|Black\right) P\left(Square|Black\right) P\left(Black\right)$$
$$= \frac{1}{3} \times \frac{2}{3} \times \frac{9}{13} = \frac{2}{13}$$

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Outline

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Now

OBSERVATIONS

- There are good savings in the Number of Values

Brute Force Approach

- on n binary variables requires m^n, if $m = \max\{|v_i| \, |V\}_{i=1}^{n}$.

For a Bayesian Network with n binary variables and each node has at most k parents

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It is more!!!

Theorem (Markov Condition on a DAG)

- Let a DAG G be given in which each node is a random variable, and let a discrete conditional probability distribution of each node given values of its parents in G be specified.
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Outline

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Example

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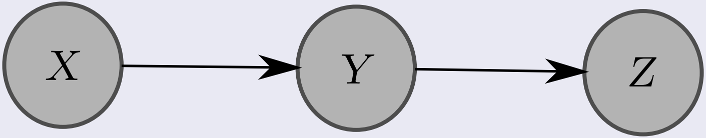

$$P(x_1) = 0.3 \qquad P(y_1|x_1) = 0.6 \qquad P(z_1|y_1) = 0.2$$
$$P(x_2) = 0.7 \qquad P(y_2|x_1) = 0.4 \qquad P(z_2|y_1) = 0.8$$

$$P(y_1|x_2) = 0.0 \qquad P(z_1|y_2) = 0.5$$
$$P(y_2|x_2) = 1.0 \qquad P(z_2|y_2) = 0.5$$

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Then

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Causality in Bayesian Networks

Definition of a Cause

The one, such as a person, an event, or a condition, that is responsible for an action or a result.

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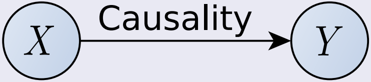
Causality in Bayesian Networks

Nevertheless, It sheds light in the issue

- If the action of making variable X take some value sometimes changes the value taken by a variable Y.



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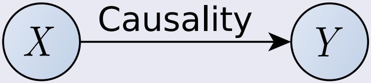

Here, we assume X is responsible for sometimes changing Y 's value

- Thus, we conclude X is a cause of Y.

Furthermore

Formally

We say we **manipulate** X when we force X to take some value.

- We say X **causes** Y if there is some manipulation of X that leads to a change in the probability distribution of Y.

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Precautionary Tale: Causality and Bayesian Networks

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- Consider the dependence between Lung Cancer, L, and the X-ray test, X.
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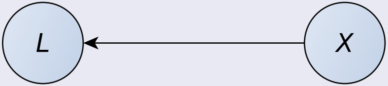
$$P(l_1) = 0.001 \qquad L \longrightarrow X \qquad \begin{array}{l} P(x_1|l_1) = 0.6 \\ P(x_1|l_2) = 0.02 \end{array}$$

Precautionary Tale: Causality and Bayesian Networks

However, we can try the same

$$P(l_1|x_1) = 0.02915$$
$$P(l_1|x_2) = 0.00041$$



$$P(x_1) = 0.02058$$

Remark

Be Careful

- It is tempting to think that Bayesian Networks can be created by creating a DAG where the edges represent direct causal relationships between the variables.

Outline

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However

Causal DAG

- Given a set of variables V, if for every $X, Y \in V$ we draw an edge from X to Y $\iff$ X is a direct cause of Y relative to V, we call the resultant DAG a **causal DAG**.

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Examples of those

- Common Causes
- Common Effects

However, we still want to know if the Markov Condition Holds

Remark

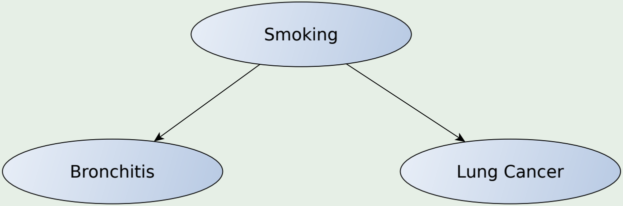
There are several thing that the DAG needs to have in order to have the Markov Condition.

Examples of those

- Common Causes
- Common Effects

How to have a Markov Assumption : Common Causes

Consider



Markov condition

$$I_p(\{B\},\{L\}\mid\{S\}) \Rightarrow P(B|L,S)=P(B|S) \quad (12)$$

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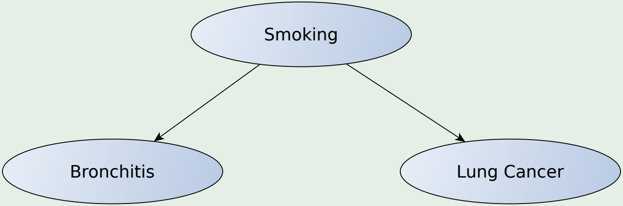

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$$S \rightarrow B \text{ and } S \rightarrow L \tag{13}$$

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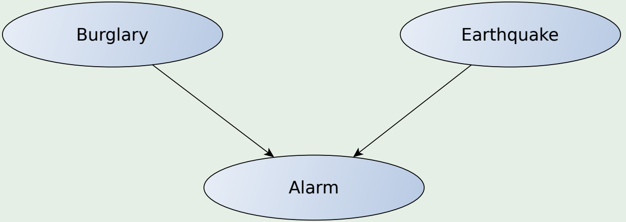

Markov Condition

$$I_p(B,W) \Rightarrow P(B|E) = P(B) \quad (14)$$

Thus

We would expect Raining and Ballgame to be independent of each other which is in agreement with the Markov condition.

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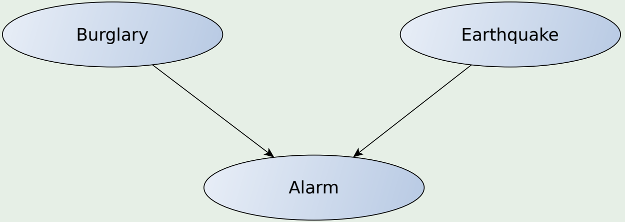

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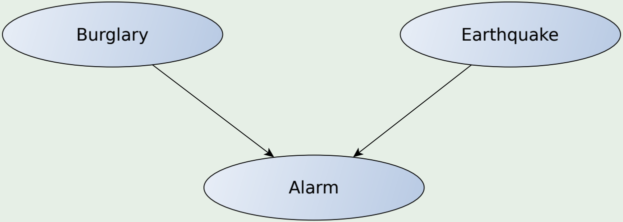

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If the alarm has gone off, news that there had been an earthquake would 'explain away' the idea that a burglary had taken place.

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Outline

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The Causal Markov Condition

What do we want?

The basic idea is that the Markov condition holds for a causal DAG.

Rules to construct A Causal Graph

Conditions

1. There must be no hidden common causes.
2. There must not be selection bias.
3. There must be no feedback loops.

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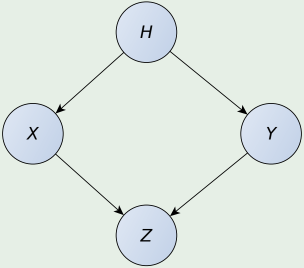
Consider the following DAG



Something Notable

- If a DAG is created on the basis of causal relationships between the variables under consideration,
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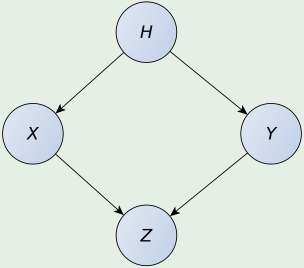

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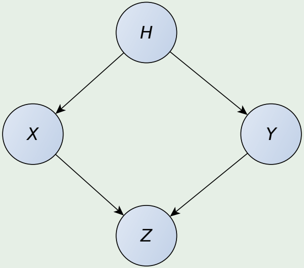

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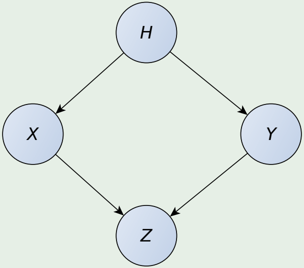

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Outline

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Inference in Bayesian Networks

What do we want from Bayesian Networks?

The main point of Bayesian Networkss is to enable probabilistic inference to be performed.

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It is used to obtain the posterior probability of one or more variables given evidence concerning the values of other variables.

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It finds the most probable configuration of a set of variables (hypothesis) given certain evidence.

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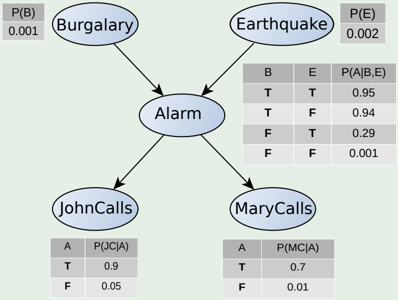

P(B)
0.001

P(E)
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B	E	P(A|B,E)
T	**T**	0.95
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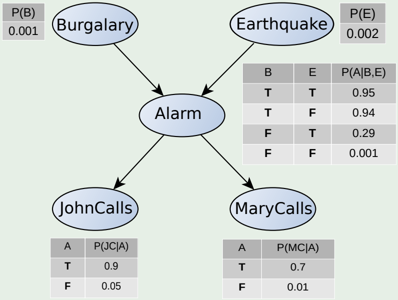

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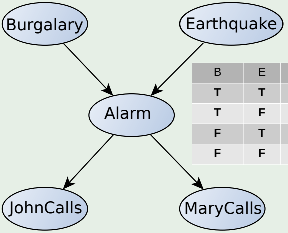

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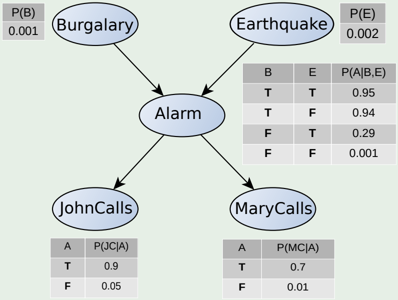

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- $P(earthquake|JCalls, MCalls)$
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Outline

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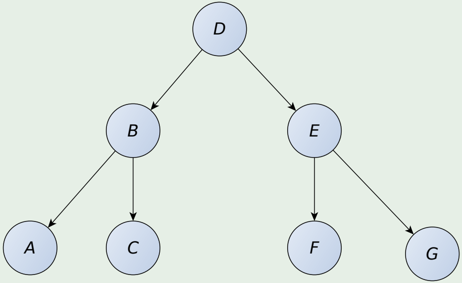
Example

DAG



We have the following model

$p(a,b,c,d,e,f,g)$ is modeled by

$$p(a,b,c,d,e,f,g) = p(a|b)\,p(c|b)\,p(f|e)\,p(g|e)\,p(b|d)\,p(e|d)\,p(d)$$

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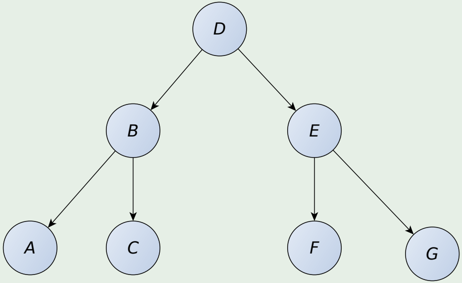

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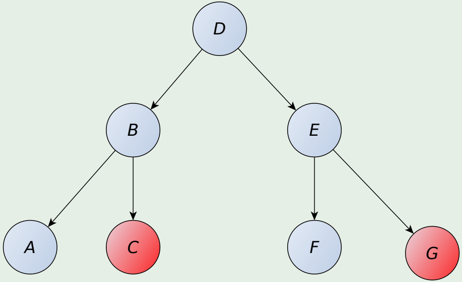

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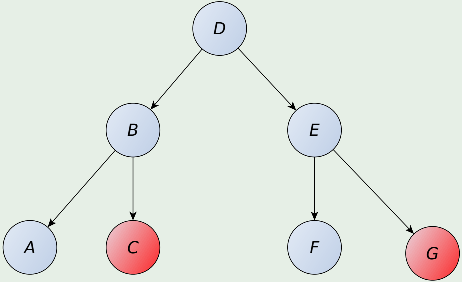

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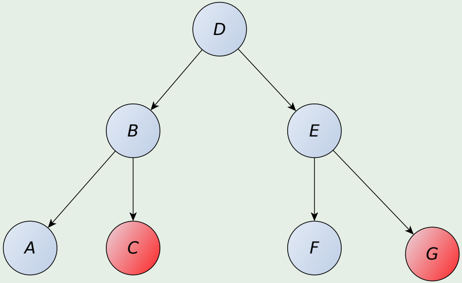
Then, if you have brute force approach



However, a direct calculation leads to use a demarginalization

$$p(a|c,g) = \sum_{b,d,e,f} p(a,b,d,e,f|c,g)$$

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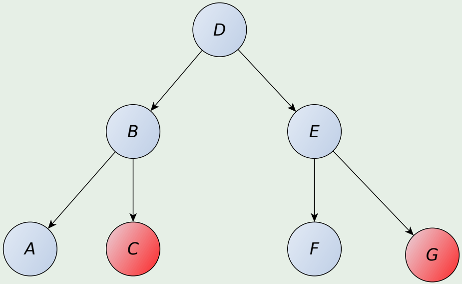

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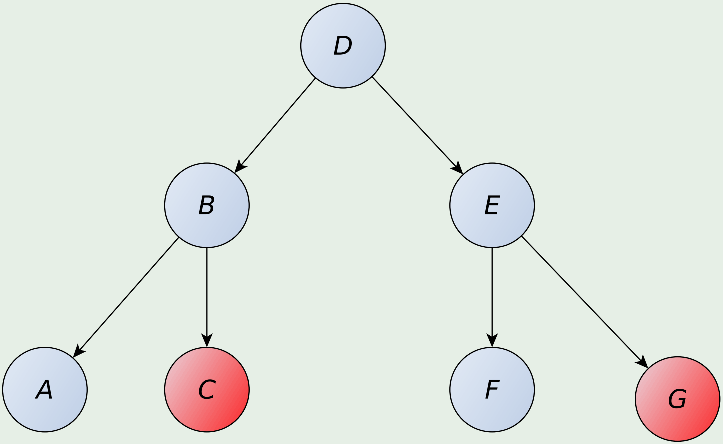
Example

We get some information about $(a = a_i, c = c_i, g = g_i)$



Thus, we have by the Markov Condition

First, we use the chain representation

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Then, we have that

Using the DAG structure

$$p\left(a=a_i,b,d,e,f|c=c_i,g=g_i\right) = p\left(a=a_i|b\right)p\left(b|d,c=c_i\right)\times\cdots$$
$$\ldots p\left(d|e\right)p\left(e,f|g=g_i\right)$$

Then given the original sum at the de-margenalization

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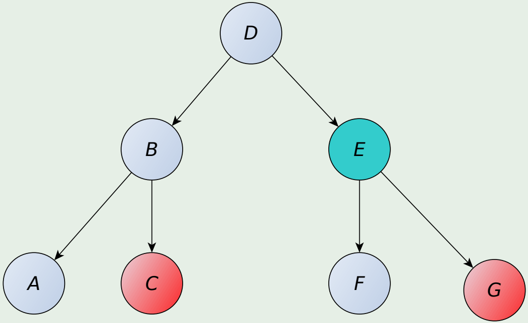
Now, using the relation with respect to E

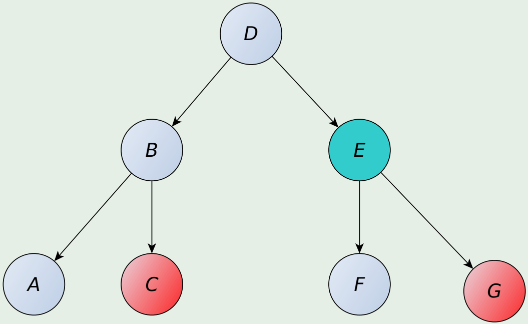


Using this information, we can reduce one of the sums by marginalization

$$\sum_f p\left(e, f|g = g_i\right) = p\left(e|g = g_i\right)$$

Now, we can concentrate $\sum_f p\left(e, f | g = g_i\right)$

Now, using the relation with respect to E



Using this information, we can reduce one of the sums by marginalization

$$\sum_f p\left(e, f | g = g_i\right) = p\left(e | g = g_i\right)$$

How?

Remember that

$$\sum_f p\left(e, f|g=g_i\right) = \sum_f p\left(e|f, g=g_i\right) p\left(f|g=g_i\right)$$

$$= \sum_f pp\left(e|f, g=g_i\right)$$

$$= p\left(e|g=g_i\right)$$

How?

Remember that

$$
\begin{aligned}
\sum_f p\left(e, f | g = g_i\right) &= \sum_f p\left(e | f, g = g_i\right) p\left(f | g = g_i\right) \\
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&= p\left(e | g = g_i\right)
\end{aligned}
$$

How?

Remember that

$$\begin{aligned}\sum_f p\left(e,f|g=g_i\right) =& \sum_f p\left(e|f,g=g_i\right)p\left(f|g=g_i\right)\\ =& \sum_f pp\left(e|f,g=g_i\right)\\ =& p\left(e|g=g_i\right)\end{aligned}$$

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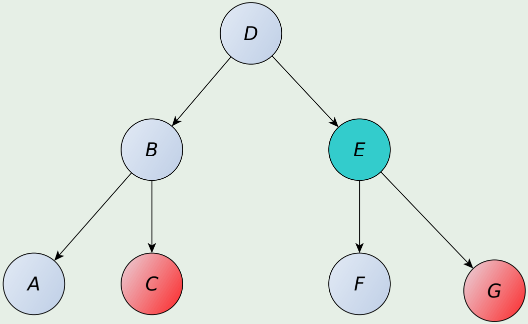
Then, we have that

DAG



Thus, we can reduce the size of our sum

$$\sum_{b} p\left(a = a_i | b\right) \sum_{d} p\left(b | d, c = c_i\right) \sum_{e} p\left(d | e\right) p\left(e | g = g_i\right)$$

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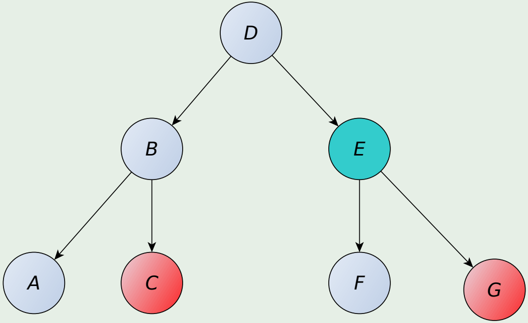
DAG



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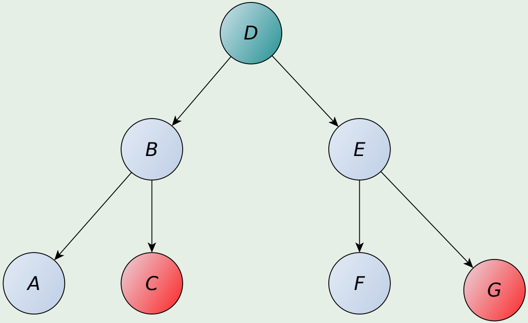
Then, we can use the realtion with respect to D

Given the following DAG



Now, we can calculate the probability of D by using the chain rule

$$p(d|e)\,p(e|g=g_i) = p(d|e,g=g_i)\,p(e|g=g_i) = p(d,e|g=g_i)$$

Then, we can use the realtion with respect to D

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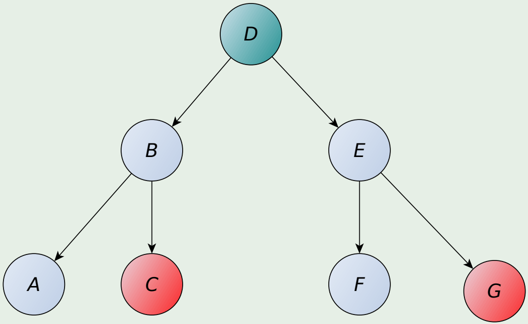
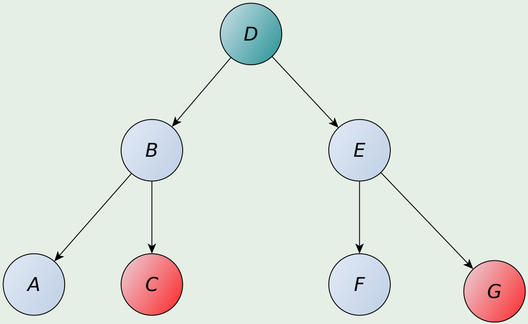


Now, we can calculate the probability of D by using the chain rule

$$p\left(d|e\right)p\left(e|g=g_i\right)=p\left(d|e,g=g_i\right)p\left(e|g=g_i\right)=p\left(d,e|g=g_i\right)$$

Example

DAG

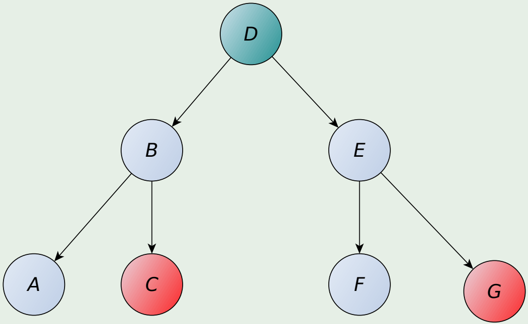


Now, we can calculate the probability of D by using the chain rule

$$\sum_{b} p\left(a = a_i | b\right) \sum_{d} p\left(b | d, c = c_i\right) \sum_{e} p\left(d, e | g = g_i\right)$$

Example

DAG



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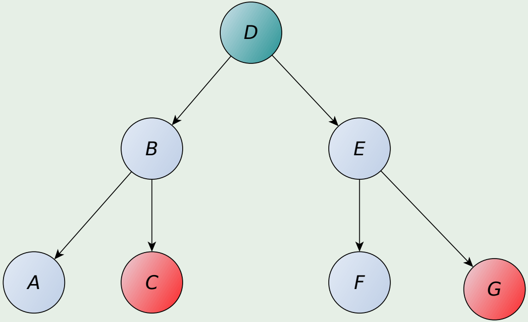
Example

DAG

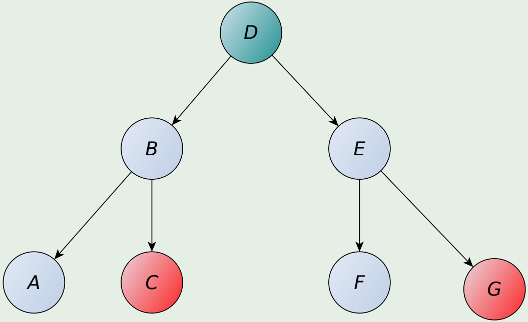


Now, we sum over all possible values of E

$$\sum_{e} p\left(d, e | g = g_i\right) = p\left(d | g = g_i\right)$$

Example

DAG



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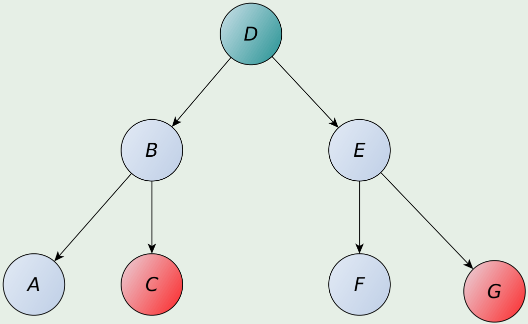
Example

DAG

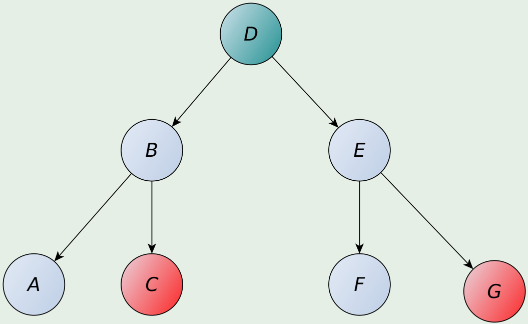


We get the following

$$\sum_b p(a = a_i|b) \sum_d p(b|d, c = c_i)\, p(d|g = g_i)$$

Example

DAG

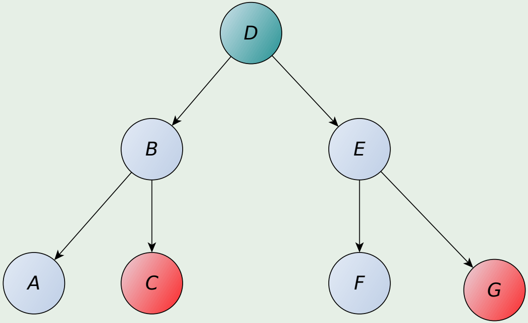


We get the following

$$\sum_{b} p\left(a = a_i | b\right) \sum_{d} p\left(b | d, c = c_i\right) p\left(d | g = g_i\right)$$

Example

DAG

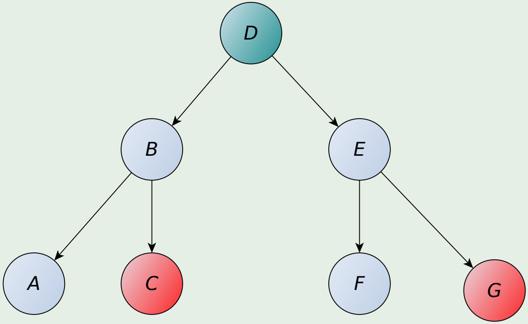


Again the chain rule for D

$$p\left(b|d, c=c_i\right) p\left(d|g=g_i\right) = p\left(b|d, c=c_i, g=g_i\right) p\left(d|c=c_i, g=g_i\right)$$
$$= p\left(b, d|c=c_i, g=g_i\right)$$

Example

DAG

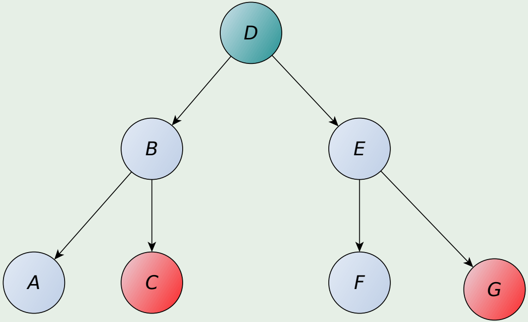


Again the chain rule for D

$$p(b|d, c = c_i)\, p(d|g = g_i) = p(b|d, c = c_i, g = g_i)\, p(d|c = c_i, g = g_i)$$
$$= p(b, d|c = c_i, g = g_i)$$

Example

DAG

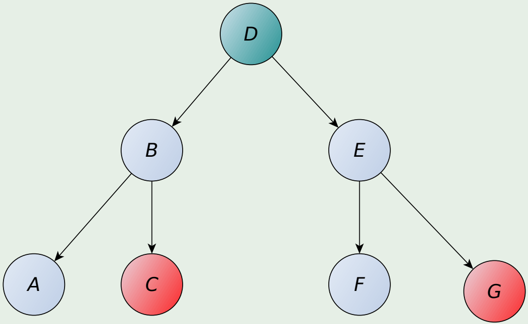


Now, we sum over all possible values of D

$$\sum_{b} p\left(a = a_i | b\right) p\left(b | c = c_1, g = g_1\right)$$

Example

DAG



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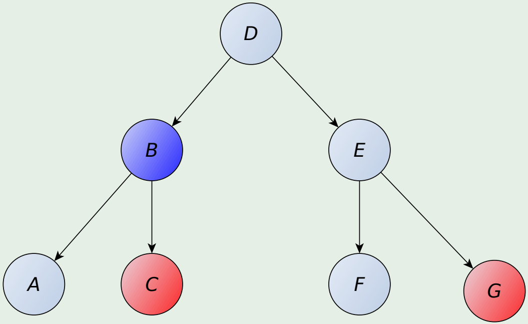
Example

DAG

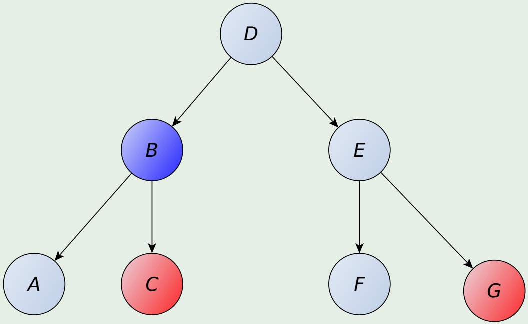


Now, we use the chain rule for reducing again

$$p(a=a_i|b)\,p(b|c=c_i,g=g_i) = p(a=a_i,b|c=c_i,g=g_i)$$

Example

DAG



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$$p\left(a=a_i|b\right)p\left(b|c=c_i,g=g_i\right)=p\left(a=a_i,b|c=c_i,g=g_i\right)$$

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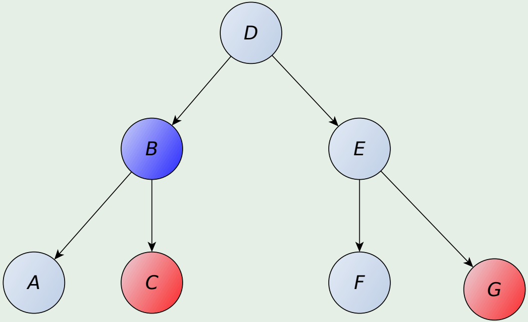
Example

DAG



Now, we use the chain rule for reducing again

$$\sum_{b} p\left(a = a_i, b | c = c_i, g = g_i\right) = p\left(a = a_i | c = c_i, g = g_i\right)$$

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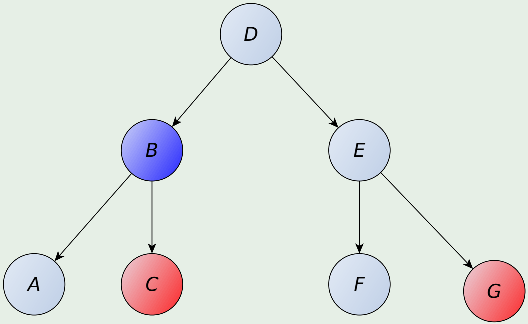

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$$\sum_{b} p\left(a = a_i, b | c = c_i, g = g_i\right) = p\left(a = a_i | c = c_i, g = g_i\right)$$

Complexity

Because this can be computed using a sequence of four for loops

The complexity simply becomes $O(m)$ when compared with $O(m^4)$

Outline

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General Strategy for Inference

Query

- Want to compute $P(q|e)$!!!

Step 1

- $P(q|e) = \frac{P(q,e)}{P(e)} = \alpha P(q,e)$, since $\alpha = P(e)$ is constant wrt Q.

Step 2

- $P(q,e) = \sum_{a..z} P(q,e,a,b,\ldots,z)$, by the law of total probability.

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- Distribute summations across product terms for efficient computation.

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  - Message passing (Pearl, 1988)

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