Introduction to Artificial Intelligence

Constraint Satisfaction Problems

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Outline

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- For path constraints, constraints refer to the path as a whole.
  - Expressed in temporal logic

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Domain of a Variable

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Thus

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Degree of a Variable

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- The degree of a variable is the number of constraints in which it is involved.

Example

$$X_1 + X_3 + 3X_2 < 15$$
$$7X_2 \times 4X_5 = 84$$
$$2X_1 + 6X_4 - X_2 \geq 9X_3$$

Then, Degree(X_1) = 2, Degree(X_2) = 3, etc

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Constraint

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- A constraint on a set of variables is a restriction on the set of values that these variables can take simultaneously.

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Constrain Satisfaction

Where is it used?

- Constraint satisfaction is used to model and solve combinatorial problems.

Something Notable

- Constraint satisfaction relies on a declarative problem description that consists of a set of variables together with their respective domains.

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- $0 \leq X \leq 9$
- $X + Y = 7$
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A constraint satisfaction problem (CSP) consists of

- A finite set of variables $V_1, ..., V_n$ over finite domains $D_{v_1}, ..., D_{v_n}$
- A finite set of constraints $C = \{C_1, ..., C_m\}$
  - They are relation between arbitrary variables
- A set $R = \{R_1, ..., R_m\}$ of m relations associated with the constraints where each one of the relations

$$R_i \in D_{i1} \times ... \times D_{ik}$$

  define all combinations of values permitted by C_i.

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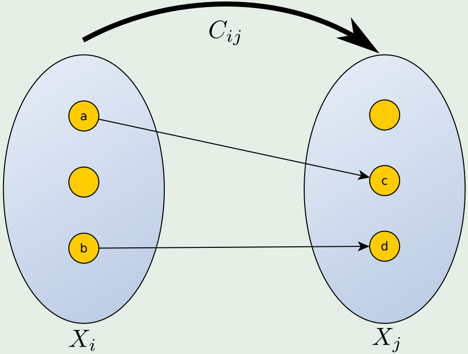
The Different Representations of a CSP

Graphical Representation

- The Most Common...
- Thus, we have two main representations

The Different Representations of a CSP

A representation via a graph local to the constraint



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Or a Global Representation

- Via a graph of al CSP constraints.

Associating

- Any CSP (X, D, C, R) a graph of constraints $G = (X, C)$whose nodes represent the variables and the edges the constraints.

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Representation in Extensions

- The set of pairs authorized for the binary constraints or more generally the n-uplets authorized for the n-ary constraints.

Representation in Intention

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Nevertheless

Finally, we want a solution

- It is a complete assignment of values to variables satisfying all the constraints.

IMPORTANT

For the sake of simplicity

- We are ruling out continuous variables in the definition!!!

Outline

Example

We have

- Binary CSP is a CSP where the constraints involve only two variables

For example take the following CSP

- $X + Y = Z,\ X < Y$
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The Final Constraints

Constraints

The constraints that induce no conflict are

- $X_i \neq X_j$ (no vertical threat) for all $1 \leq i \neq j \leq 8$
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Definition

You need to color a map with k colors in such a way that the two neighboring areas, having a common border, are not of the same color.

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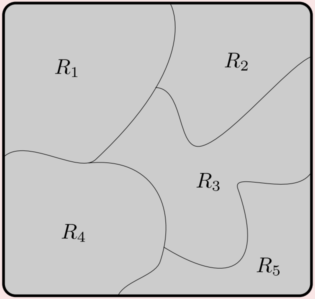

The CSP for the Map-Coloring

You have the triplet (X, D, C)

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How do we solve this?

Using an efficient Algorithm

Such a problem formulation calls for an efficient search algorithm to find a feasible variable assignment representing valid placements of the queens on the board.

Naive solution

A naive strategy considers all 8^8 possible assignments, which can easily be reduced to $8!$.

Better

We need a refined approach maintains a vector for a partial assignment in a vector, which grows with increasing depth and shrinks with each backtrack.

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This is known as consistency

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Simple version

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Procedure Consistent

Input: Label set L, constraints C

Output: L satisfies C true/false

1. **for each** $c \in C$
2. **if** $Variables(c) \subseteq L$
3. **if not** $Satisfied(c, L)$
4. **return false** // *When an inconsistency happens*
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- $Variables\,(c)$ denotes the set of variables in mentioned in the constraint c.
- $Satisfied(c, L)$ to denote if the constraint c is satisfied by the current label set L(values to variables).

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We need something better

There is a long list of algorithms for this.

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Basic Definitions

Definition - K-Consistency

A CSP (X, D, C, R) is k-consistent if and only if, for any n-tuplet of k variables $(X_1, ..., X_k)$ of X, any consistent $k-1$ instantiation may be extended to a consistent instantiation with the k^{th} variable.

Definition - Strong K-Consistency

A CSP $P(X, D, C, R)$ is said to be strongly k-consistent if and only if, $\forall i, 1 \leq i \leq k$, P is i-consistent.

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Basic Definitions

Definition - Node Consistency

A node-consistent CSP (X, D, C, R) is a 1-consistent CSP. This consistency is only verified if for any X_i variable of X, and for any v_i value of D_i, the partial assignment (X_i, v_i) satisfies all the unary constraints of C involving this variable.

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Definition

A CSP (X, D, C, R) is called ***arc consistent*** if and only if, for any couple of variables (X_i, X_j) of X, each couple represents an arc in the associated constraint graph, and for any value v_i from the domain D_i that satisfies the unary constraints involving X_i, there is a value v_j in the domain D_j compatible with v_i.

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Consider a simple CSP with variables A and B

- Domains $D_A = \{1, 2\}$ and $D_B = \{1, 2, 3\}$
- With constraint $A < B$

Thus...

We can easily remove 1 from D_B.

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- Given a planar graph, assign one of 4 colors to each vertex such that any two adjacent vertices have different colors.

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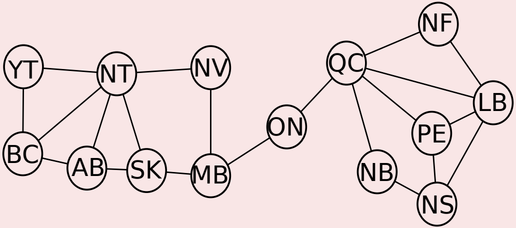

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It is applied to

Couple of variables (X_i, X_j) connected by a constraint C_{ij} by removing the locally inconsistent values from the X_i.

- The arc consistency is verified if and only if all the arcs on the graph of constraints are arc consistent.

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- AC-3
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Couple of variables (X_i, X_j) connected by a constraint C_{ij} by removing the locally inconsistent values from the X_i.

Where

- This couple of variables represents an arc in the graph of constraints often denoted by (i, j).
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Two Main Algorithms

We will look at

- AC-1
- AC-3
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Revise Procedure

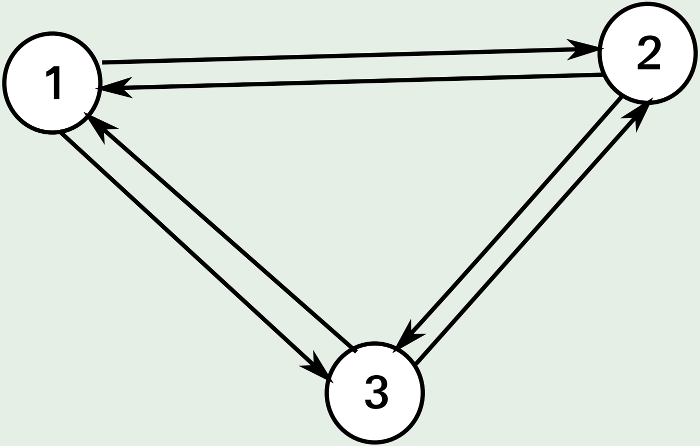
Revise(i, j)

output Boolean

1. $CHANGE = False$
2. **for each** $x \in D_i$
3. **If there is no** $y \in D_j$ **such that** $R_{ij}(x, y)$ **is true then**
4. **delete** x **from** D_i
5. $CHANGE = True$
6. **return** $CHANGE$

Example we are going to use

Graph to color



Outline

AC-1

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- Early version is due to ROSENFELD A., HUMMEL R., ZUCKER S.

- (X_i, X_j) are linked by a constraint C_{ij}
- The algorithm visits each couple (X_i, X_j) and removes all the values that violate C_{ij} from domain D_i

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AC-1 Algorithms

Procedure AC1

1. $Q = \{(i,j) | C_{ij} \in C, i \neq j\}$.
2. **Repeat**
3. $\quad CHANGE = False$
4. $\quad$ **for each** $(i,j) \in Q$ **do**
5. $\quad\quad CHANGE = (Revise(i,j) \vee CHANGE)$
6. **Until** $\neg Change$

Example

We have that

- In the first column, the current couple of variables (i, j) being treated by the revise procedure is colored in **red**.
- In the second column, the value (color) removed by the revise procedure is colored in **red**.
- In the second column, the final domains obtained after performing the whole AC-1 are emphasized.

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Example

Then

ITERATION	Q	D_i	Change
1(Repeat).1(For)	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1 = \{R,G,B\}$ $D_2 = \{R,G\}$ $D_3 = \{G\}$	FALSE
1.2	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1 = \{R,G,B\}$ $D_2 = \{R,G\}$ $D_3 = \{G\}$	FALSE
1.3	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1 = \{R,G,B\}$ $D_2 = \{R,G\}$ $D_3 = \{G\}$	TRUE

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1.3	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1=\{R,G,B\}$ $D_2=\{R,G\}$ $D_3=\{G\}$	TRUE

Example

Then

ITERATION	Q	D_i	Change
1.4	$\{(1,2)\,;(2,1);(1,3)$ $(3,1);(2,3)\,;(3,2)\}$	$D_1=\{R,B\}$ $D_2=\{R,G\}$ $D_3=\{G\}$	TRUE
1.5	$\{(1,2)\,;(2,1);(1,3)$ $(3,1)\,;(2,3);(3,2)\}$	$D_1=\{R,B\}$ $D_2=\{R,G\}$ $D_3=\{G\}$	TRUE
1.6	$\{(1,2)\,;(2,1);(1,3)$ $(3,1)\,;(2,3)\,;(3,2)\}$	$D_1=\{R,B\}$ $D_2=\{R\}$ $D_3=\{G\}$	TRUE

Example

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Example

Then

ITERATION	Q	D_i	Change
2.1	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1=\{R,B\}$ $D_2=\{R\}$ $D_3=\{G\}$	TRUE
2.2	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1=\{B\}$ $D_2=\{R\}$ $D_3=\{G\}$	TRUE

Example

Then

ITERATION	Q	D_i	Change
2.1	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1 = \{R, B\}$ $D_2 = \{R\}$ $D_3 = \{G\}$	TRUE
2.2	$\{(1,2);(2,1);(1,3)$ $(3,1);(2,3);(3,2)\}$	$D_1 = \{B\}$ $D_2 = \{R\}$ $D_3 = \{G\}$	TRUE

Complexity of AC-1

Something Notable

The systematic nature of the revisions of all the problem's arcs, each time a value is removed, deteriorates the AC-1 performances, since the removal of a value from a variable X.

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We do the following

- The algorithm uses a queue structure Q.
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At each step a pop element, an arc (i,j), is revised using the **Revise** procedure.

During this revision

If the removal of a value v_i occurs in D_i, the set of arcs (k,i) such as $k \neq i$ and $k \neq j$ are added to Q (If not there already).

Next

AC-3 re-examines the viability of all the values $v_k \in D_k$ in relation to C_{ki}.

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AC-3 re-examines the viability of all the values $v_k \in D_k$ in relation to C_{ki}.

Important!!!

We can have the following

It may happen that (i, v_i) is the sole support for certain values of D_k and the removal of v_i makes them not arc consistent.

Outline

Algorithm AC-3

Procedure AC-3

Input: Set of variables V, set of domains D, set of constraints C

Output: Satisfiable true/false, restricted set of domains

1. Q=$\{(i,j)\,|c_{ij} \in C, i \neq j\}$ **// Where Q is a Queue**
2. **while** $(Q \neq \emptyset)$
3. $c = Q.pop()$
4.   **if Revise**(i,j)
5. $Q = Q \cup \{(k,i)\,|c_{ki} \in C, k \neq i, k \neq j\}$

Explanation

Then

1. All the arcs are added to the queue Q.
2. If an arc has been removed... it can be added again to Q **(Line 5)**
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- For a complete graph of constraints, the Q will see the insertion of $O\left(n^2 d\right)$ arcs
- Temporal Complexity $O\left(n^2 d^3\right)$ - More efficient than AC-3
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Example

Then

Iteration	Q	D_i	Revise (i, j)
1	{(1, 2); (2, 1); (1, 3); (3, 1); (2, 3); (3, 2)}	D1 = {R, G, B} D2 = {R, G} D3 = {G}	FALSE
2	{(2, 1); (1, 3); (3, 1); (2, 3); (3, 2)}	D1 = {R, G, B} D2 = {R, G} D3 = {G}	FALSE
3	{(1, 3); (3, 1); (2, 3); (3, 2)}	D1 = { R, G, B} D2 = {R, G} D3 = {G}	TRUE
4	{(3, 1); (2, 3); (3, 2) ∪ **(2, 1)**}	D1 = {R, B} D2 = {R, G} D3 = {G}	FALSE

Example

Then

5	{(2, 3); (3, 2); (2, 1)}	D1 = {R, B} D2 = {R, G} D3 = {G}	TRUE
6	{(3, 2); (2, 1) ∪ **(1, 2)}**	D1 = {R, B} D2 = {R} D3 = {G}	FALSE
7	{(2, 1); (1, 2)}	D1 = {R, B} D2 = {R} D3 = {G}	FALSE
8	{(1, 2)}	D1 = {R, B} D2 = {R} D3 = {G}	TRUE
9	{∅ ∪ **(3, 1)**}	D1 = {B} D2 = {R} D3 = {G}	FALSE

There are other methods for arc consistency

You can look at them in

“Constraint Satisfaction Problems: CSP Formalisms and Techniques” by Khaled Ghedira.

Outline

How can we use this?

Various techniques for solving CSP have been developed

Classification of them:

- Complete methods that guarantee completeness (quality) at the expense of efficiency (temporal complexity)
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We will look at a complete resolution method: Backtracking

- Remember it?? Solving NP-Problems

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Backtracking Algorithm

BackTracking(V, I)

```
If V = ∅ then
        I is a solution
else
        Let x ∈ V
        for each v ∈ D_x do
                If I ∪ {(x, v)} is consistent then
                        BackTracking (V − {x} , I ∪ (x, v))
```

Outline

Example

Pruning Example

Given the possible values that you can give to two literals:

x_1	x_2
1	1
1	0
0	1
$\mathbf{0}$	$\mathbf{0}$

It is possible to prune a quarter of the entire search space... Can this be systematically exploited?

An example of exploiting this idea in SAT solvers

Consider the following Boolean formula $\phi(w, x, y, z)$

$$(w \vee x \vee y \vee z) \wedge (w \vee \neg x) \wedge (x \vee \neg y) \wedge (y \vee \neg z) \wedge (z \vee \neg w) \wedge (\neg w \vee \neg z)$$

We start branching in one variable, we can choose w

Note: This selection does not violate any of the clauses of $\phi(w, x, y, z)$

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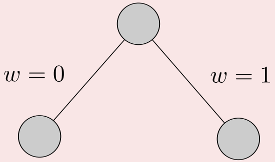
Initial formula ϕ



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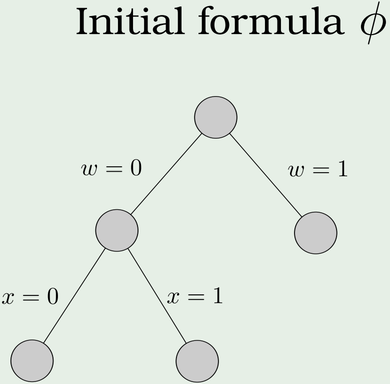
Now

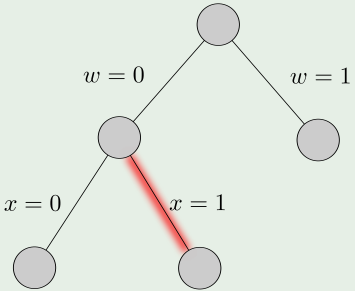
The partial assignment $w = 0, x = 1$ violates the clause $(w \vee \neg x)$



Now

Then, we prune that branch

Initial formula ϕ



In addition

What if $w = 0, x = 0$

instantiation

Then, the following clauses are satisfied

1. $\neg w = 1$
2. $\neg x = 1$

Thus, we have the following left

1. Before
   1. $(w \vee x \vee y \vee z) \wedge (w \vee \neg x) \wedge (x \vee \neg y) \wedge (y \vee \neg z) \wedge (z \vee \neg w) \wedge (\neg w \vee \neg z)$
2. After
   1. $(0 \vee 0 \vee y \vee z) \wedge (0 \vee 1) \wedge (0 \vee \neg y) \wedge (y \vee \neg z) \wedge (z \vee 1) \wedge (1 \vee \neg z)$

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   1. $(0 \vee 0 \vee y \vee z) \wedge (0 \vee 1) \wedge (0 \vee \neg y) \wedge (y \vee \neg z) \wedge (z \vee 1) \wedge (1 \vee \neg z)$

Finally

We have the following reduced number of equations

$$(y \vee z), (1), (\neg y), (y \vee \neg z), (1), (1) \Leftrightarrow \boldsymbol{(y \vee z), (\neg y), (y \vee \neg z)}$$

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2. After
   1. $(1) \wedge (0) \wedge (1) \wedge (y \vee \neg z) \wedge (1) \wedge (1)$

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2. After
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Thus

We have something no satisfiable

$(1) \wedge (0) \wedge (1) \wedge (y \vee \neg z) \wedge (1) \wedge (1) \Leftrightarrow (), (y \vee \neg z)$

Clearly

We prune that part of the search tree.

Note: we use "()≡(0)" to point out to a "empty clause" ruling out satisfiability.

Thus

We have something no satisfiable

$(1) \wedge (0) \wedge (1) \wedge (y \vee \neg z) \wedge (1) \wedge (1) \Leftrightarrow (), (y \vee \neg z)$

Clearly

We prune that part of the search tree.

Note we use "$()\equiv(0)$" to point out to a "empty clause" ruling out satisfiability.

The decisions we need to make in backtracking

First

Which subproblem to expand next.

Second

Which branching variable to use.

Remark

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1. **Failure**: the subproblem has no solution.
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Example

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